From Analysis to Surface: Generating the Surface of Milton Babbitt's *Sheer Pluck* from a Parsimonious Encoding of an Analysis of its Pitch Structure

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Intention

- Generate the surface of *Sheer Pluck* from a encoding of an analysis its pitch structure.

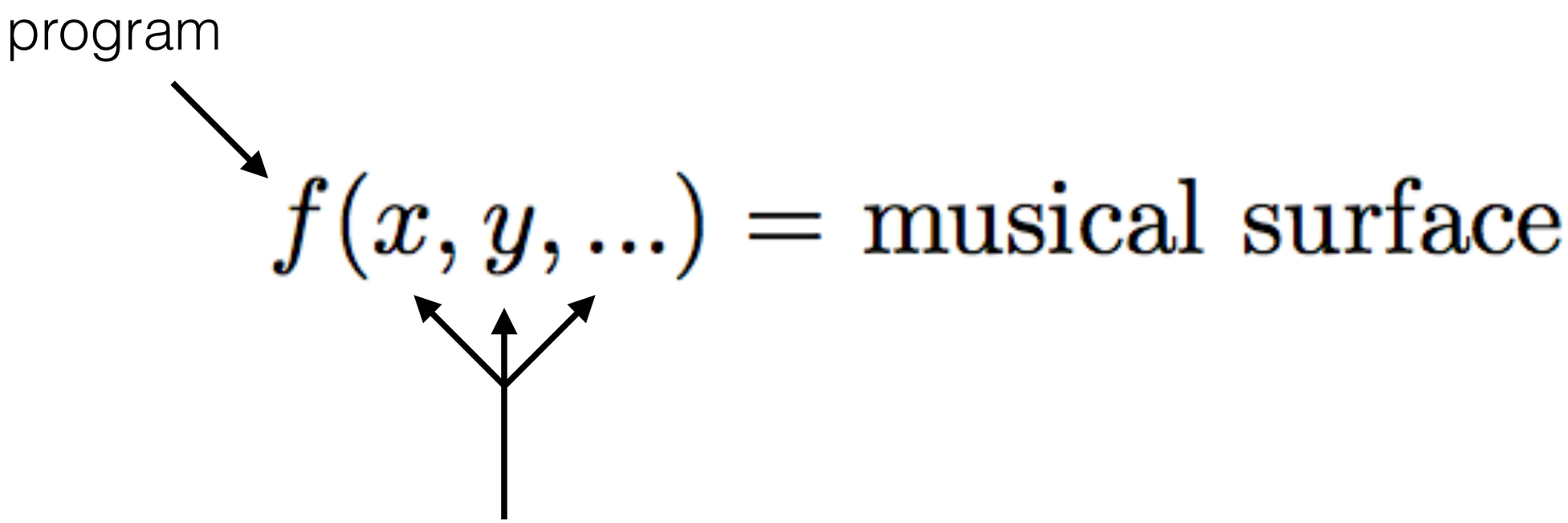
Analysis ⟶ Surface

compare with

Zip file ⟶ uncompressed file

- Generating the surface (“unziping” the analysis) proves no loss of information, i.e. the analysis contains all the information in the surface.

input and output



input: the smallest amount of information possible

Overview

1. Describe the basic properties of a six-part all-partition array.

2. Provide a template for and list of constraints on the organization of *Sheer Pluck's* pitch structure.

    2.1. Organize 48 twelve-tone rows into *hexachordally combinatorial* pairs.

3. Demonstrate the computational issues involved in automatically parsing *Sheer Pluck*.

    3.1. Parse the resultant structure into 58 distinct *integer compositions*.

1. Background

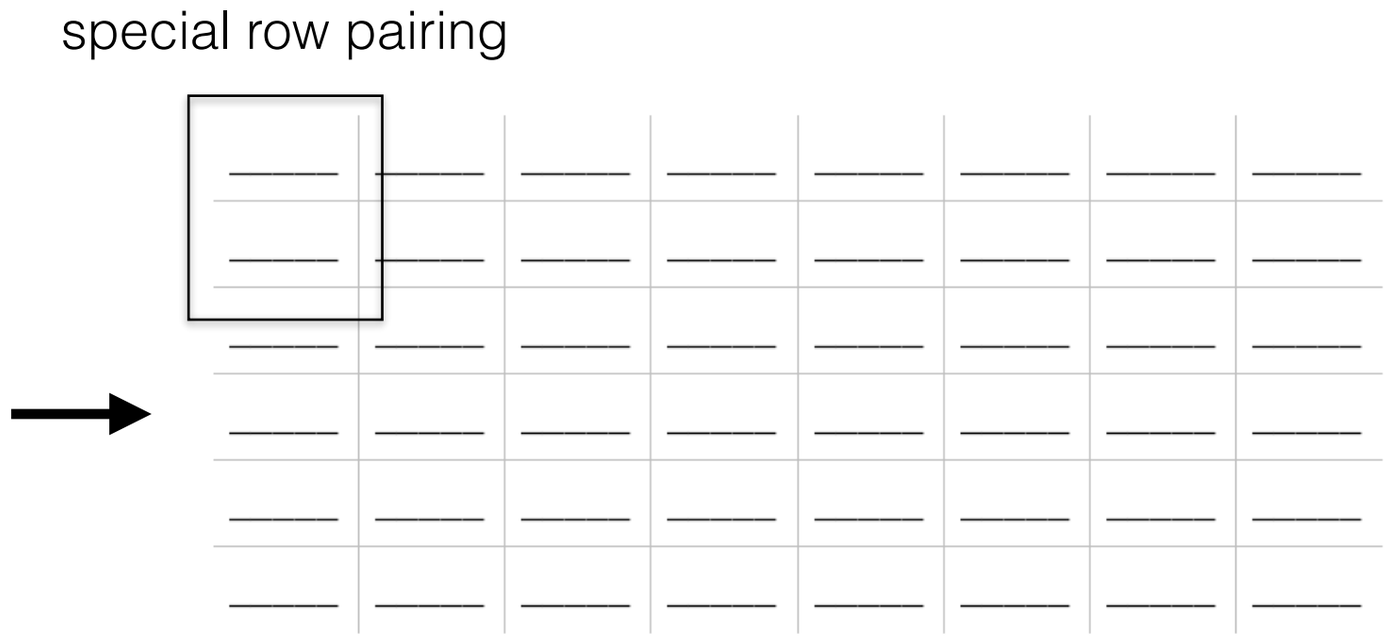
- What is an all-partition array?

Lyne $(x, \overline{x})$	2 / 3 10 11 9		9
Lyne $(\overline{x}, x)$	1 8		8
Lyne $(y, \overline{y})$	0	0 11 1 5	6
Lyne $(\overline{y}, y)$	7 /	10 9 4 2 8 3 6 7	7
Lyne $(z, \overline{z})$	6 5		5 0 10 4 11 2 3
Lyne $(\overline{z}, z)$	4 /		1
	$52^2 1^3$	84	71^5

…a twelve-tone structure organized into pairs of *hexachordally combinatorial* rows and then parsed into a sequence of discrete, vertical aggregates by distinct *integer compositions*.

Babbitt Square Row Organization

	I0	I5	I4	I6	I10	I11	I2	I9	I3	I1	I8	I7	
P0	0	5	4	6	10	11	2	9	3	1	8	7	R0
P7	7	0	11	1	5	6	9	4	10	8	3	2	R7
P8	8	1	0	2	6	7	10	5	11	9	4	3	R8
P6	6	11	10	0	4	5	8	3	9	7	2	1	R6
P2	2	7	6	8	0	1	4	11	5	3	10	9	R2
P1	1	6	5	7	11	0	3	10	4	2	9	8	R1
P10	10	3	2	4	8	9	0	7	1	11	6	5	R10
P3	3	8	7	9	1	2	5	0	6	4	11	10	R3
P9	9	2	1	3	7	8	11	6	0	10	5	4	R9
P11	11	4	3	5	9	10	1	8	2	0	7	6	R11
P4	4	9	8	10	2	3	6	1	7	5	0	11	R4
P5	5	10	9	11	3	4	7	2	8	6	1	0	R5
	RI0	RI5	RI4	RI6	RI10	RI11	RI2	RI9	RI3	RI1	RI8	RI7	



- 48 equivalent rows under P, I, R and RI organized into a **6**X8 grid = **six** part all-partition array
- Similarly, these 48 rows organized into a **4**X12 grid = **four**-part all-partition array

Row Pairing

$$\overbrace{\qquad\qquad\qquad\qquad}^{\{x\}}$$

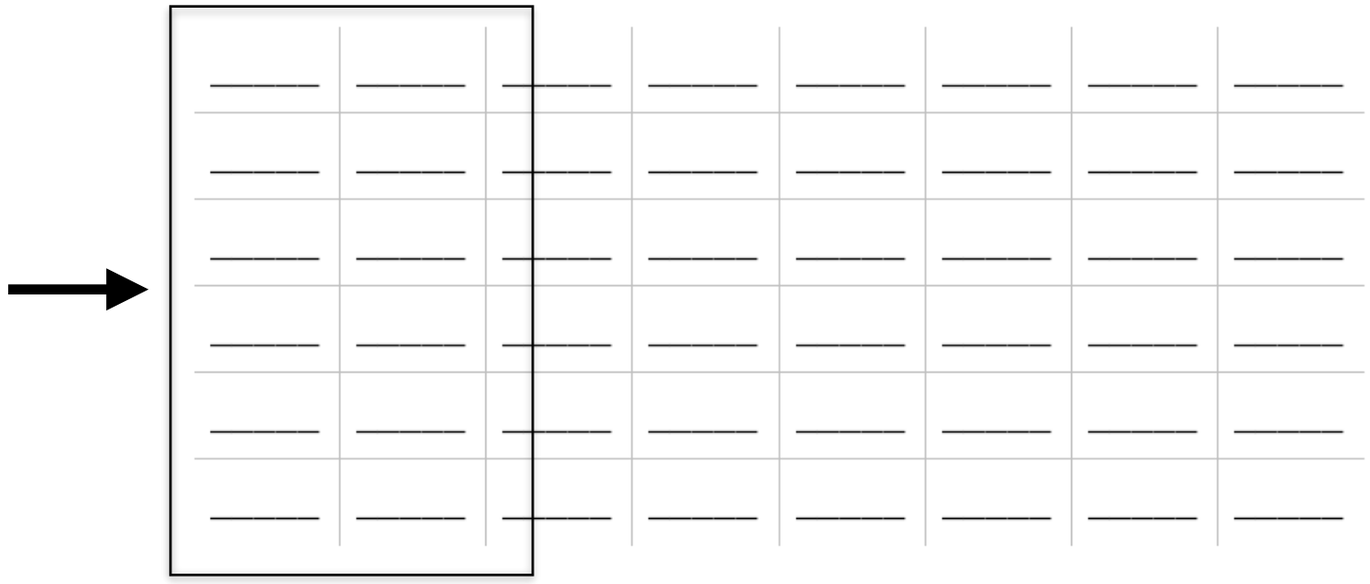
Row $(x, \overline{x})$	$(11, 4, 3, 5, 9, 10, 1, 8, 2, 0, 7, 6)$
Row $(\overline{x}, x)$	$(6, 7, 0, 2, 8, 1, 10, 9, 5, 3, 4, 11)$

$$\underbrace{\qquad\qquad\qquad\qquad}_{\{x\}}$$

- Based on the principle of *hexachordal combinatoriality*.
- *h*-related rows form both linear and vertical aggregates, four in total.

Babbitt Square Row Organization continued…

	I0	I5	I4	I6	I10	I11	I2	I9	I3	I1	I8	I7	
P0	0	5	4	6	10	11	2	9	3	1	8	7	R0
P7	7	0	11	1	5	6	9	4	10	8	3	2	R7
P8	8	1	0	2	6	7	10	5	11	9	4	3	R8
P6	6	11	10	0	4	5	8	3	9	7	2	1	R6
P2	2	7	6	8	0	1	4	11	5	3	10	9	R2
P1	1	6	5	7	11	0	3	10	4	2	9	8	R1
P10	10	3	2	4	8	9	0	7	1	11	6	5	R10
P3	3	8	7	9	1	2	5	0	6	4	11	10	R3
P9	9	2	1	3	7	8	11	6	0	10	5	4	R9
P11	11	4	3	5	9	10	1	8	2	0	7	6	R11
P4	4	9	8	10	2	3	6	1	7	5	0	11	R4
P5	5	10	9	11	3	4	7	2	8	6	1	0	R5
	RI0	RI5	RI4	RI6	RI10	RI11	RI2	RI9	RI3	RI1	RI8	RI7	



Blocks and Row Types

	Block I	Block II	
	$\overbrace{}^{\{x\}}$	$\overbrace{}^{\{x\}}$	
Lyne $(x, \overline{x})$	$(11, 4, 3, 5, 9, 10, 1, 8, 2, 0, 7, 6)$	$(5, 4, 11, 9, 3, 10, 1, 2, 6, 8, 7, 0)$	...
Lyne $(\overline{x}, x)$	$(6, 7, 0, 2, 8, 1, 10, 9, 5, 3, 4, 11)$	$(0, 7, 8, 6, 2, 1, 10, 3, 9, 11, 4, 5)$	
	$\underbrace{}_{\{y\}}$	$\underbrace{}_{\{y\}}$	
Lyne $(y, \overline{y})$	$(5, 6, 11, 1, 7, 0, 9, 8, 4, 2, 3, 10)$	$(1, 0, 7, 5, 11, 6, 9, 10, 2, 4, 3, 8)$	...
Lyne $(\overline{y}, y)$	$(2, 9, 10, 8, 4, 3, 0, 5, 11,1, 6, 7)$	$(4, 9, 8, 10, 2, 3, 6, 1, 7, 5, 0, 11)$	
	$\overbrace{}^{\{z\}}$	$\overbrace{}^{\{z\}}$	
Lyne $(z, \overline{z})$	$(0, 5, 4, 6, 10, 11, 2, 9, 3, 1, 8, 7)$	$(4, 5, 10, 0, 6, 11, 8, 7, 3, 1, 2, 9)$	...
Lyne $(\overline{z}, z)$	$(1, 8, 9, 7, 3, 2, 11, 4, 10, 0, 5, 6)$	$(3, 2, 9, 7, 1, 8, 11, 0, 4, 6, 5, 10)$	

- A *row type* refers to a row's hexachord content. Concatenated rows (often of the same type) are called **lynes**.

- The columns of the 6X8 grid are called **blocks**.

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✓ 1. Describe the basic properties of a six-part all-partition array.

⇨ 2. Provide a template for and list of constraints on the organization of *Sheer Pluck's* pitch structure.

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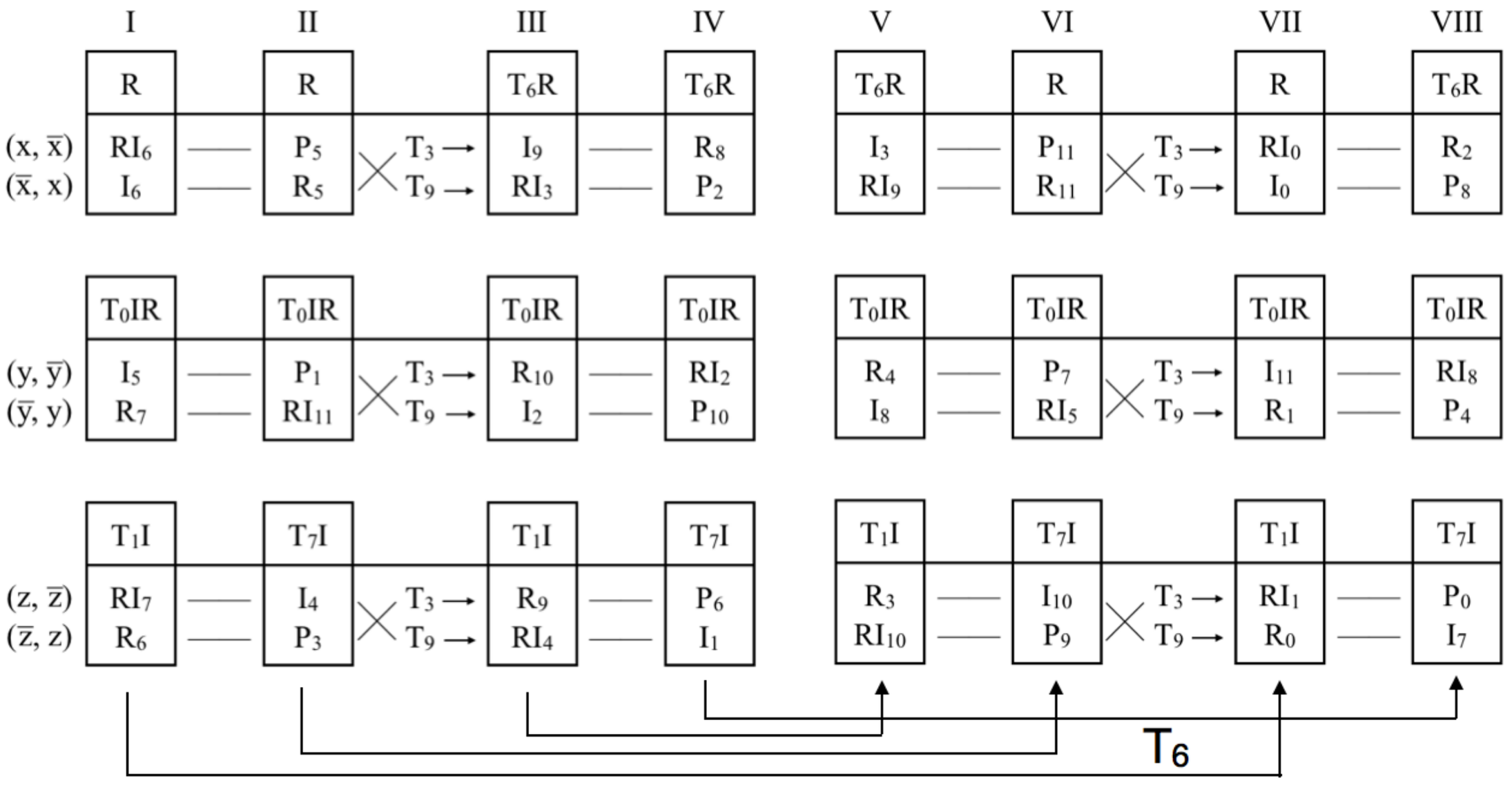
3. Demonstrate the computational issues involved in automatically parsing *Sheer Pluck*.

3.1. Parse the resultant structure into 58 distinct integer compositions.

2.1. Row Template of Babbitt's *Sheer Pluck*



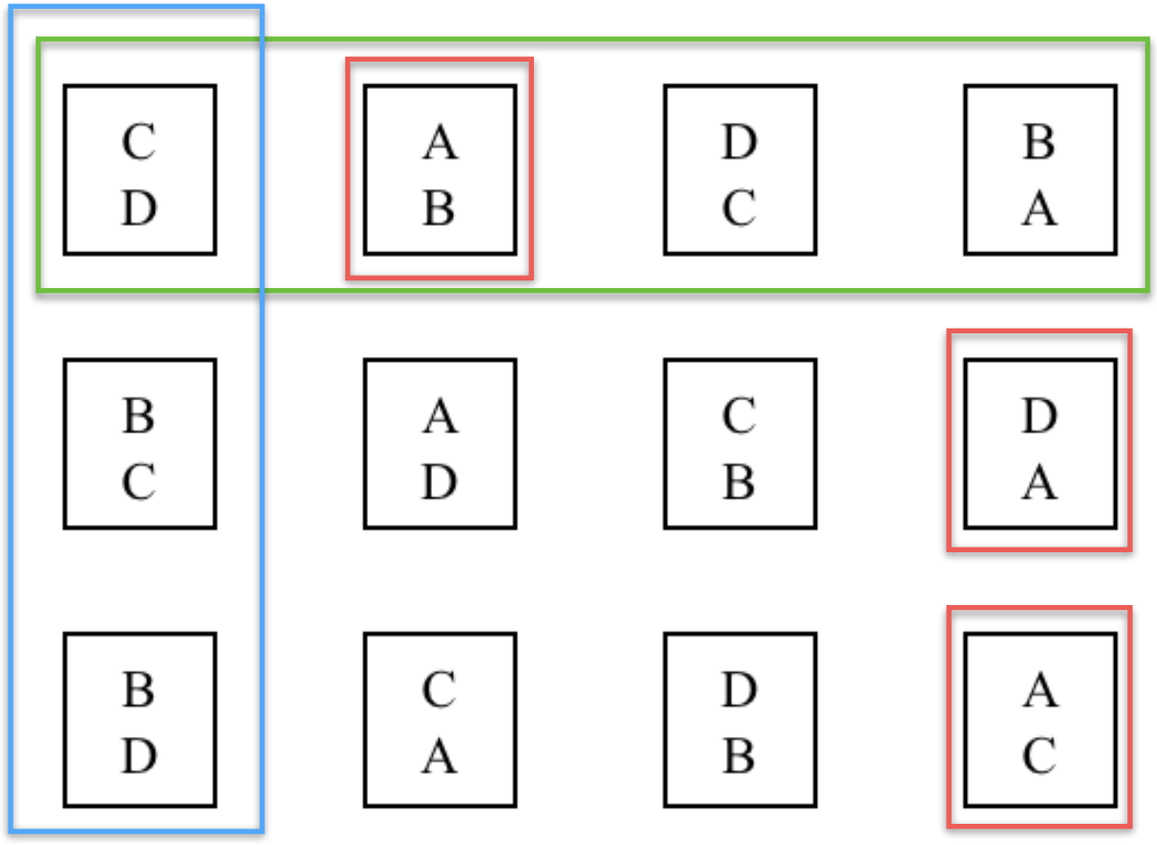
Complement Transformations, T_3 and T_9

Where

$$\{A, B, C, D\} = \{P, I, R, RI\}$$

Any given pair of *h*-related rows, x and y…

$$\{\{x, y\} : x \neq y \wedge \{x, y\} \subset \{A, B, C, D\}\}$$



Any given pair of h-related rows in a single lyne pair, x? and y?…

$$\{\{x_0, y_0\} : x_0 \neq y_0 \wedge (x_0 \in \{A, B\}, y_0 \in \{A, B\}) \cap (x_0 \in \{C, D\}, y_0 \in \{C, D\})\}$$

$$\{\{x_1, y_1\} : x_1 \neq y_1 \wedge (x_1 \in \{A, D\}, y_1 \in \{A, D\}) \cap (x_1 \in \{B, C\}, y_1 \in \{B, C\})\}$$

$$\{\{x_2, y_2\} : x_2 \neq y_2 \wedge (x_2 \in \{A, C\}, y_2 \in \{A, C\}) \cap (x_2 \in \{B, D\}, y_2 \in \{B, D\})\}$$

Any given block of *h*-related rows, p and q…

$$\{\{p, q\} : p \neq q \wedge p = \{x_0, y_0\} \cup \{x_1, y_1\} \cup \{x_2, y_2\} \not\ni A, q = \{x_0, y_0\} \cup \{x_1, y_1\} \cup \{x_2, y_2\}\}$$

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3.1. Parsing the Pitch-class Structure

- Automating the organization of pitch structure is relatively straightforward. Parsing it however, is not a computationally trivial problem to solve.

- Some definitions…

Integer Compositions

- In number theory, an **integer composition** is a way of representing an integer *n* as an *ordered* sum of positive integers.

When *n* = 12

$$3+3+2+2+1+1 \neq 2+3+2+1+3+1$$

- In an all-partition array, we must include zero in many integer compositions. We call such instances **weak integer compositions**.

When *n* = 12

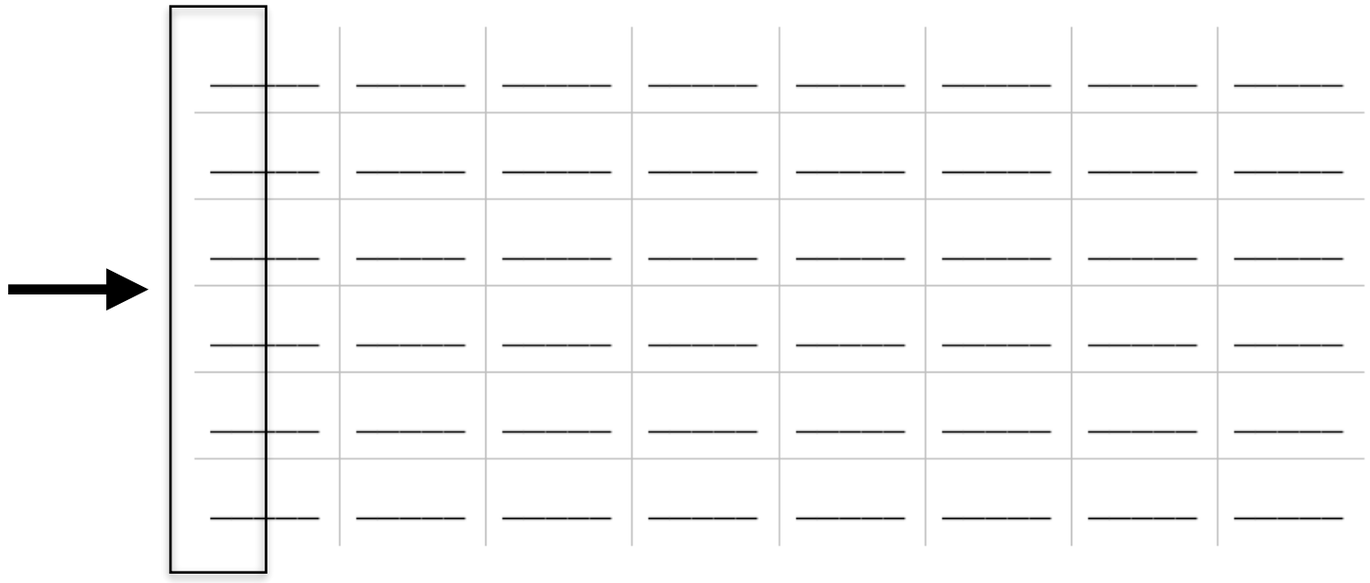
$$6+6+0+0+0+0 \neq 0+6+0+0+6+0$$

- There are 6,308 *ordered* ways of representing 12 in 6 parts, but only 58 un*ordered* ways.

- A six-part all-partition array is constructed of a distinct sequence of 58 compositions for which there are 58! possibilities.

some compositions in a block…

	I0	I5	I4	I6	I10	I11	I2	I9	I3	I1	I8	I7	
P0	0	5	4	6	10	11	2	9	3	1	8	7	R0
P7	7	0	11	1	5	6	9	4	10	8	3	2	R7
P8	8	1	0	2	6	7	10	5	11	9	4	3	R8
P6	6	11	10	0	4	5	8	3	9	7	2	1	R6
P2	2	7	6	8	0	1	4	11	5	3	10	9	R2
P1	1	6	5	7	11	0	3	10	4	2	9	8	R1
P10	10	3	2	4	8	9	0	7	1	11	6	5	R10
P3	3	8	7	9	1	2	5	0	6	4	11	10	R3
P9	9	2	1	3	7	8	11	6	0	10	5	4	R9
P11	11	4	3	5	9	10	1	8	2	0	7	6	R11
P4	4	9	8	10	2	3	6	1	7	5	0	11	R4
P5	5	10	9	11	3	4	7	2	8	6	1	0	R5
	RI0	RI5	RI4	RI6	RI10	RI11	RI2	RI9	RI3	RI1	RI8	RI7	



A portion of a single parsed block…

11 4 3 5		9 10 1 8 2	…
6 7	0 2		…
5	6 11 1	7	…
2 9	10 8 4 3	0 5 11	…
0	5	4 6	…
1 8	9 7	3	…

$$Compositions = \left| \begin{matrix} (4,2,1,2,1,2) \\ (0,2,3,4,1,2) \\ (5,0,1,3,2,1) \end{matrix} \right| \left| \begin{matrix} \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \end{matrix} \right| = 58\, compositions$$

A problem…

$$58 \cdot 12 = 696$$

$= 576\,pcs$

$+120\,pcs$

n distinct objects into k distinct boxes where *k* = 48 and *n* = 120

$$48^{120} = 5.61 \cdot 10^{201}$$

$$\approx 1 \cdot 10^{80}$$

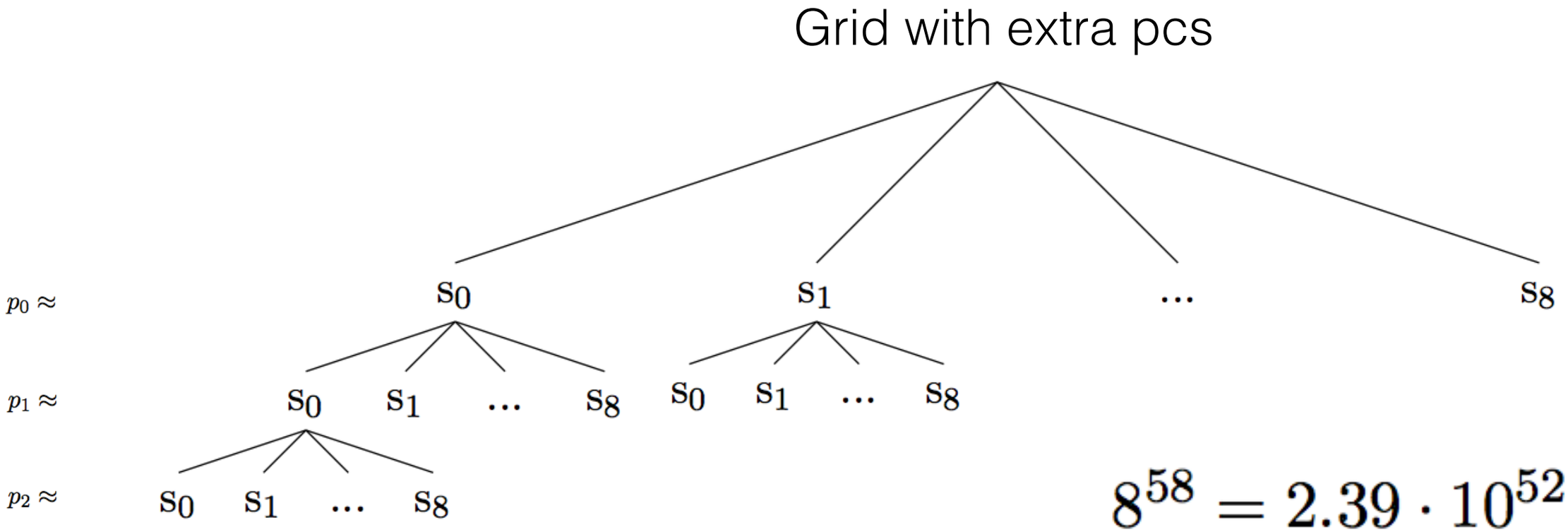
A portion of a single parsed block with extra pcs…

2 3 10 11 **9**		**9**	…	= 118
7 **8**		**8**	…	= 117
0	**0** 11 1 5	6	…	= 111
1	10 9 4 2 8 3 6 **7**	**7**	…	= 114
6 **5**		**5** 0 10 4 11 2 3	…	= 118
4		1	…	= 118

= 696

$(5, 2, 1, 1, 2, 1)$ $(0, 0, 4, 8, 0, 0)$ $(1, 1, 1, 1, 7, 1)$

Brute force search for possible successful integer compositions in *Sheer Pluck*



Where p_x is the ordinal position of a given composition and *s* is a successful integer composition.

Conclusion…

$$f(\text{musical surface}) = \text{musical surface}$$

- Alternatively, give as input the sequence of compositions.

Questions for Future Research

- Are there additional constraints in the pitch-class structure that will limit the number of calculations required in finding successful integer compositions?

  – Yes, there must be…Likely a relationship between repeated pcs and compositions. Greedy algorithm, heurstics?

- MEI – Database of not just Babbitt's works but analyses as well?

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