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TOWARD A FORMAL COGNITIVE THEORY OF HARMONIC PITCH STRUCTURE IN THE MUSIC OF J.S. BACH

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1. Pitch class simultaneity representations and the aim of the theory.

A. Pitch simultaneity representations.

A pitch simultaneity can be defined as a segment of a score which has constant pitch content throughout its extent and which differs in pitch content from neighbouring segments. A pitch simultaneity can be represented by a pitch set and the location in the score at which that pitch simultaneity begins. A score can therefore be transformed by an effective procedure into a pitch simultaneity representation. Figure 1. (a) shows the pitch simultaneity representation of the example in Figure 1.(b).

B. Pitch class simultaneity representations.

A pitch class simultaneity can be defined as a segment of a score which has constant pitch class content throughout its extent and which differs in pitch class content from neighbouring segments. A pitch class simultaneity can be represented as a pitch class set and the location in the score at which the pitch class simultaneity begins. The pitch class simultaneity representation of a score can be derived by an effective procedure from the pitch simultaneity representation by unifying any adjacent pitch simultaneities with the same pitch class content. Figure 2.(a) shows the pitch class simultaneity representation of the example in Figure 2.(b).

C. The aim of the theory.

The aim of the theory is to devise a formal rule system capable of describing the pitch class simultaneity representations of all and only well-formed Bach *Chorales*. If this can be achieved, the rule system will constitute a formal theory of pitch class structure in Bach *Chorales*. If this has really been achieved, it

(a)

1 0/3	(c',e',g')
1 1/3	(c'',e',g')
1 2/3	(f',a',c'')

(b)

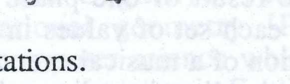

Figure 1. Pitch simultaneity representations.

(a)

1 0/3	(C,E,G)
1 2/3	(F,A,C)

(b)

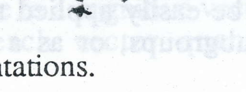

Figure 2. Pitch class simultaneity representations.

will be possible to embody the theory in two computer programs:

1. a program capable of analysing any real *Chorale* in such a way as to show how it is accountable for by the formal rule system for pitch class simultaneity structure; and
2. a program capable of generating *all and only* well-formed pitch class simultaneity structures of the *Chorale* type.

II. Path-type pitch class sets.

A. Thirds space.

Thirds space is a two-dimensional space best conceived as covering the surface of a torus. There are 12 discrete points in this space corresponding to the 12 octave equivalence pitch classes in a 12-fold equal-tempered pitch system. Each point in the space has four other points which are proximal to it in the space. For a pitch class, p, these four points represent the pitch classes, (p + 4) mod 12, (p - 4) mod 12, (p + 3) mod 12 and (p - 3) mod 12. Thirds space is isomorphic to the direct product of the two cyclic groups, C3 and C4 (Balzano, 1980).

B. Definition of path-type pitch class sets.

Pitch class sets in thirds space can be conceived in two different ways - one "dynamic" and one "static".

The "static" conception involves describing regions or areas of the space which contain all the pitch classes in the set which it is necessary to represent. This leads to the representation of the diatonic set as a "most compact region" of the space containing seven different pitch classes (Balzano, 1980). When this mode of conception is adopted, the diatonic set has unique properties which it does not share with any other seven-member pitch class set.

The "dynamic" mode of conception involves moving through the space collecting pitch classes. When this mode of representation is adopted, a special type of seven-member pitch class set can be defined. If a path is taken through thirds space such that:

1. exactly one complete circuit is made around the major circumference of the torus;
2. exactly one complete circuit is made around the minor circumference of the torus; and
3. no pitch class is passed through more than once until the starting pitch class is again reached on completion of both circuits described in 1 and 2;

then only four distinct transpositional equivalence types of pitch class set are generated and only three distinct inversional and transpositional equivalence types of pitch class set are generated. Pitch class sets which fall into these categories are called path-type sets in this study.

The minor harmonic, minor melodic ascending and major scales represent three of the four transpositional equivalence path-type sets and all of the inversional and transpositional equivalence path-type sets. The fourth transpositional equivalence type set can be represented by a "major harmonic" scale, (0,2,4,5,7,8,11), or as an inversion of the minor harmonic scale.

This would therefore seem to provide a formal definition of the category of pitch class sets which can provide the pitch class substrate of all the standard tonal scales which have emerged in music theory from an intuitive study of tonal music.

C. Path-type pitch class sets as pitch class simultaneity transition supersets.

There is no formal or computable theory of how "scales" are manifested in the structure of tonal music although many partial "theories" have emerged (for example, Rameau, 1771; Schenker, 1935/79; Lerdahl and Jackendoff, 1983). On studying the pitch class simultaneity representations of several Bach *Chorales* it has been possible to posit two hypotheses concerning the manner in which path-type pitch class sets appear to constrain the pitch structure of the style of music represented by the *Chorale* genre.

If sets S_1, S_2, S_3 and S_4 are the sets corresponding to the first four pitch class simultaneities in a piece then for each pitch class simultaneity transition, *

$$S_n \rightarrow \cancel{n+1}\ S_{n+1}$$

a pitch class simultaneity transition set can be defined, *

$$TS_n \overset{=}{+} S_n \cup S_{n+1}.$$

Each pitch class simultaneity, S_n, will be involved in two such transitions (unless it is the first or last simultaneity in the piece), TS_n and TS_{n-1}. The first path-type set structure hypothesis is that for each pitch class simultaneity, S_n, at least one of TS_n, TS_{n-1} will be a subset of at least one path-type set. The second path-type set structure hypothesis is that it is possible to interpret the path-type set structure of a well-formed *Chorale* in such a way that it is never necessary to change the operational path-type set by more than one pitch class.

III. Harmonic cores and their roots.

A. Well-formed harmonic cores and their roots.

It is useful to define a well-formed harmonic core type set as a set which is transpositionally or inversionally equivalent to the pitch class set (C,E flat, G) - that is, all major and minor triads. It is also useful to define the root of an harmonic core so that in the transpositional equivalence classes represented by the sets, (0,3,7) and (0,4,7) the root in each case is 0.

B. Three levels of pitch class simultaneity superset.

It is useful to define three levels of pitch class simultaneity superset:

1. the pitch class simultaneity set itself (simultaneity set, S);
2. the smallest connected region along the operational path-type set which contains the pitch class simultaneity set (smallest connected superset, SCS); and
3. if the SCS contains less than two harmonic cores, a number of implied supersets forming connected regions along the operational path-type set which are formable from the SCS by extension of it by only one member along the operational path-type set in either direction and only then if by doing so, the number of harmonic cores in the new set is greater than in the SCS (extended connected superset, ECS).

C. The implied harmonic cores of a pitch class simultaneity set.

For each level of pitch class simultaneity superset, a number of implied harmonic cores can be defined. These are those core-type subsets of the pitch class simultaneity superset at any one of the three levels. A loose hypothesis can be formulated as follows: that the "stability" of a pitch class simultaneity is inversely related to the number of harmonic cores which are strongly implied by the pitch class simultaneity set. Another loose hypothesis can be formulated as follows: that for a given S, the harmonic cores implied by the S are more strongly implied than those implied by the possible SCSs which are in turn more strongly implied than those implied by the possible ECSs.

D. The need for well-formedness rules for the harmonic core structure of Bach Chorales.

It is my belief that the foregoing path-type set structure theory needs to be supplemented by a system of rules constraining the harmonic core root structure of a *Chorale* in order that the aim of the theory may be achieved. I have devised several such rule systems and tested them on real *Chorales* with varying success. In two cases, a complete harmonic core root structure theory has been formulated which in combination with the foregoing path-type set structure theory is capable of providing a complete generative pathway to the pitch class simultaneity structure of all the test *Chorales*. However, earlier stages in this generative pathway do not seem to give rise necessarily to perceptually more salient aspects of the structure and so I have found it necessary to discard these theories and continue the search for a rule system capable of describing the harmonic core root structure of the *Chorale* type pitch class simultaneity representation.

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