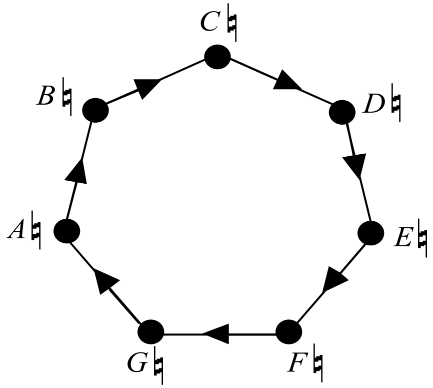
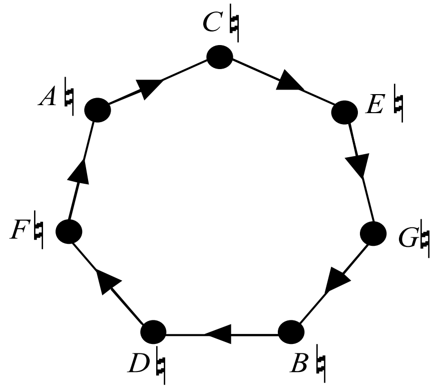
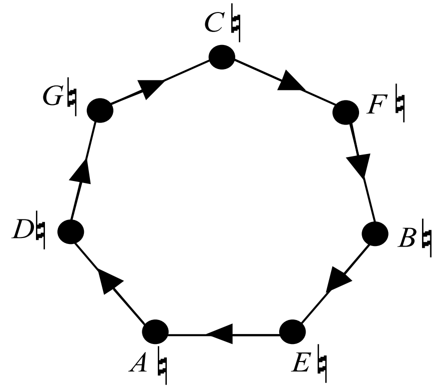


Representing pitch relations using digraphs

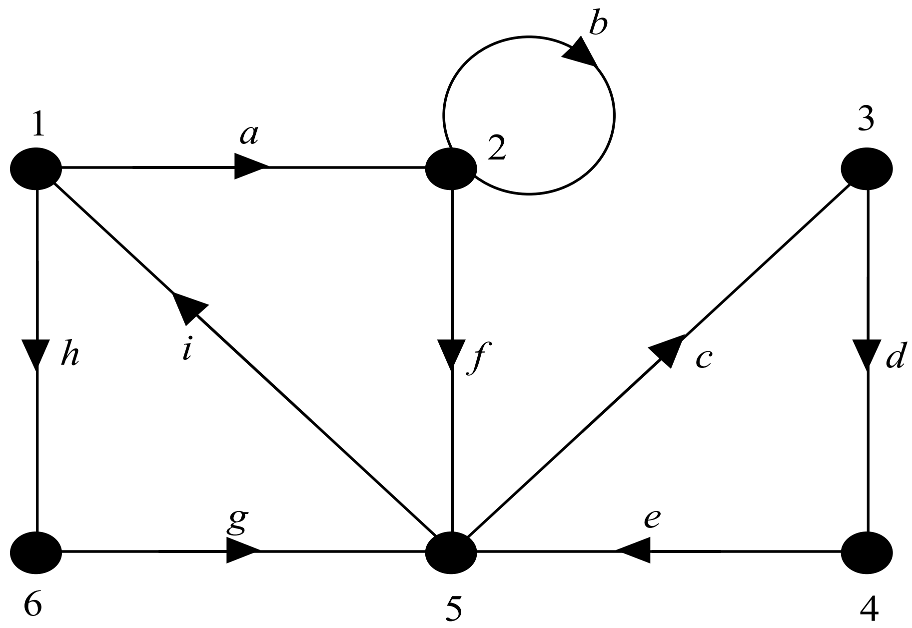


Tonal scales
(diatonic major, harmonic and melodic minor)
have special properties



DIATONIC scale is VERY special indeed!

DIGRAPHS



<u>*Walk*</u>

Any journey whatsoever

1-2-2-5

1-2-5-1-6-5-3-4-5-3

<u>*Trail*</u>

No arc more than once

1-2-5-1-6

<u>*Closed trail*</u>

No arc more than once

Begins and ends on same vertex

5-1-6-5-3-4-5

<u>*Path*</u>

No vertex more than once

1-2-5-3-4

3-4-5-1-2

<u>*Circuit*</u>

No vertex more than once

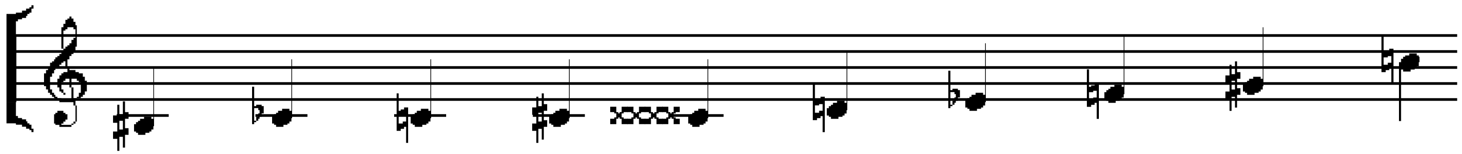
Except begins and ends on same vertex

1-6-5-1

2-2

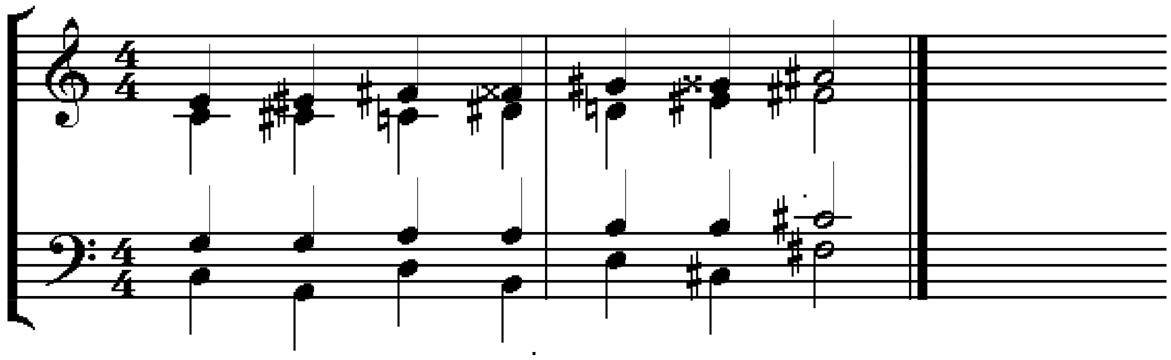
5-3-4-5

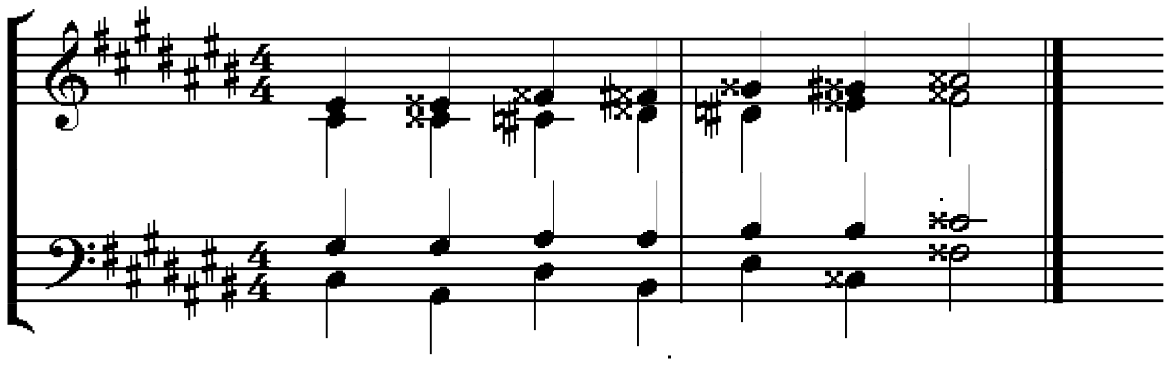
REPRESENTING PITCH



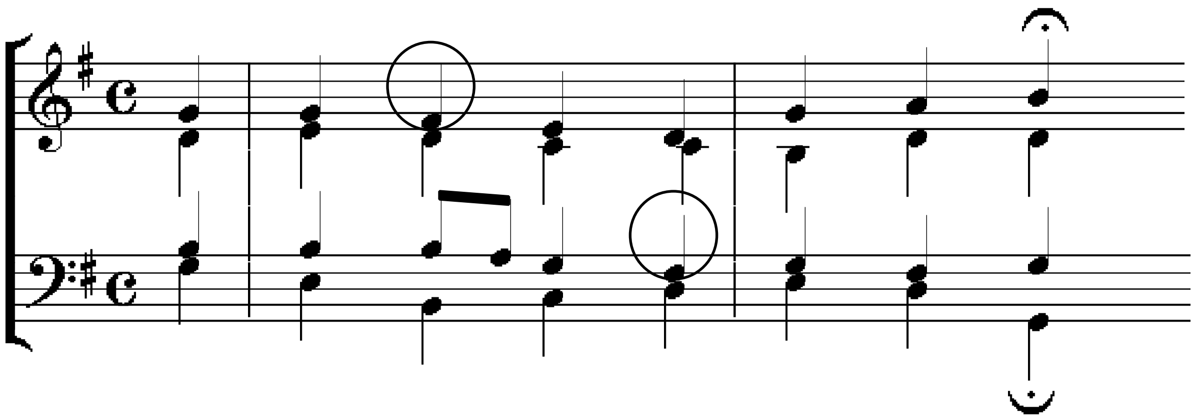
Pitch name:	$B\sharp_3$	$C\flat_4$	$C\natural_4$	$C\sharp_4$	$C{\times}{\times}{\times}{\times}_4$	$D\natural_4$	$E\flat_4$	$F\natural_4$	$G\sharp_4$	$C\natural_5$
Letter name:	B	C	C	C	C	D	E	F	G	C
Morph:	1	2	2	2	2	3	4	5	6	2
Genus name:	$B\sharp$	$C\flat$	$C\natural$	$C\sharp$	$C{\times}{\times}{\times}{\times}$	$D\natural$	$E\flat$	$F\natural$	$G\sharp$	$C\natural$
Genus:	(1,1)	(2,-1)	(2,0)	(2,1)	(2,8)	(3,0)	(4,-1)	(5,0)	(6,1)	(2,0)
Chroma:	3	2	3	4	11	5	6	8	11	3
Chromatic pitch:	39	38	39	40	47	41	42	44	47	51
Morphetic pitch:	22	23	23	23	23	24	25	26	27	30
Pitch:	(39,22)	(38,23)	(39,23)	(40,23)	(47,23)	(41,24)	(42,25)	(44,26)	(47,27)	(51,30)

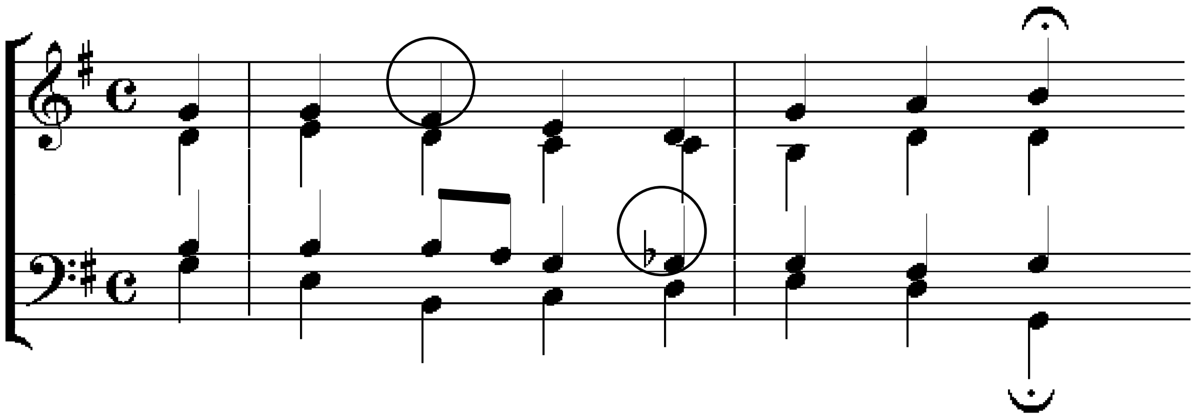
MULTIPLE SHARPS AND FLATS





LETTER-NAMES





PITCH INTERVALS

PITCH INTERVAL GENERIC EQUIVALENCE CLASSES

pu
(0,0)

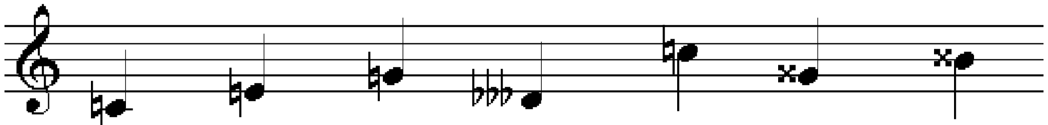
rdd2
(-1,1)

rp5
(7,4)

raa7
(13,6)

rma3
(4,2)

raa5
(9,4)



$\mathrm{C}\natural_4$ $\mathrm{E}\natural_4$ $\mathrm{G}\natural_4$ $\mathrm{D}\flat\flat\flat_4$ $\mathrm{C}\natural_5$ $\mathrm{G}\text{𝄪}_4$ $\mathrm{B}\text{𝄪}_4$

(39,23) (43,25) (46,27) (38,24) (51,30) (48,27) (52,29)

PITCH INTERVALS

rma3
(4,2)

fdd4
(-3,-3)

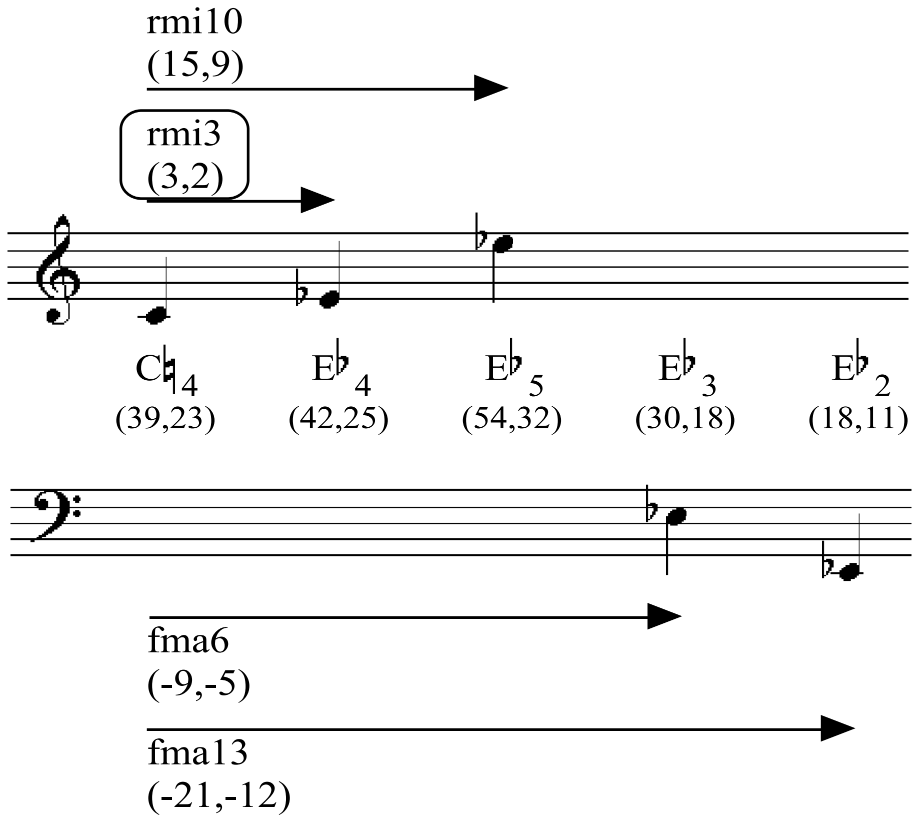
rp5
(7,4)

fdd2
(1,-1)

rdd2
(-1,1)

rp8
(12,7)

PITCH INTERVAL GENERIC EQUIVALENCE CLASSES



Two pitch intervals, *i1* and *i2*, are generically equivalent if the genus of the pitch you end up with when you transpose a pitch by *i1* is the same as when you transpose the pitch by *i2*.

PITCH RELATIONS

$$R^{\mathrm{p}} = (I\,,\,P)$$

R^{p}	PITCH RELATION
I	PITCH INTERVAL SET
P	PITCH SET

$p_1\, R^{\mathrm{p}}\, p_2$ is true if and only if
1. p_1 and p_2 are both in P
2. the interval from p_1 to p_2 is in I

EXAMPLE: if $R^{\mathrm{p}} = (\{(4,2),(3,2)\}, P^{\mathrm{u}})$

then:

$\mathrm{C}\natural_4\, R^{\mathrm{p}}\, \mathrm{E}\natural_4$ is true

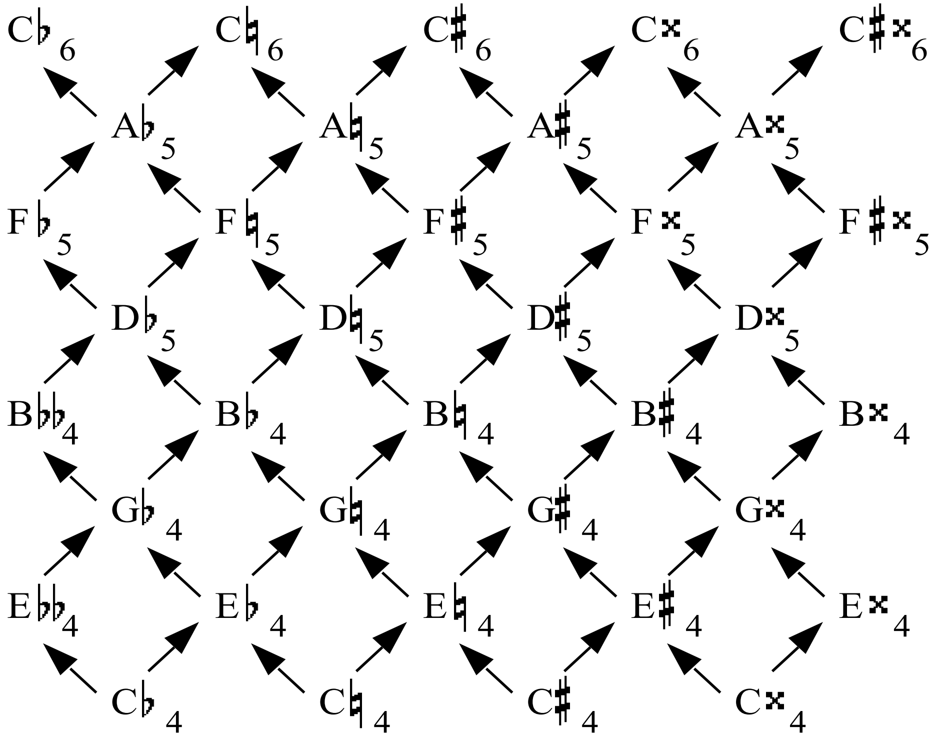
$\mathrm{C}\natural_4\, R^{\mathrm{p}}\, \mathrm{E}\flat_4$ is true

$\mathrm{C}\natural_4\, R^{\mathrm{p}}\, \mathrm{A}\flat_3$ is false

$\mathrm{C}\natural_4\, R^{\mathrm{p}}\, \mathrm{A}\natural_3$ is false

REPRESENTING PITCH RELATIONS USING DIGRAPHS

$$R^{\mathrm{p}} = (\{(4,2),(3,2)\}, P^{\mathrm{u}})$$



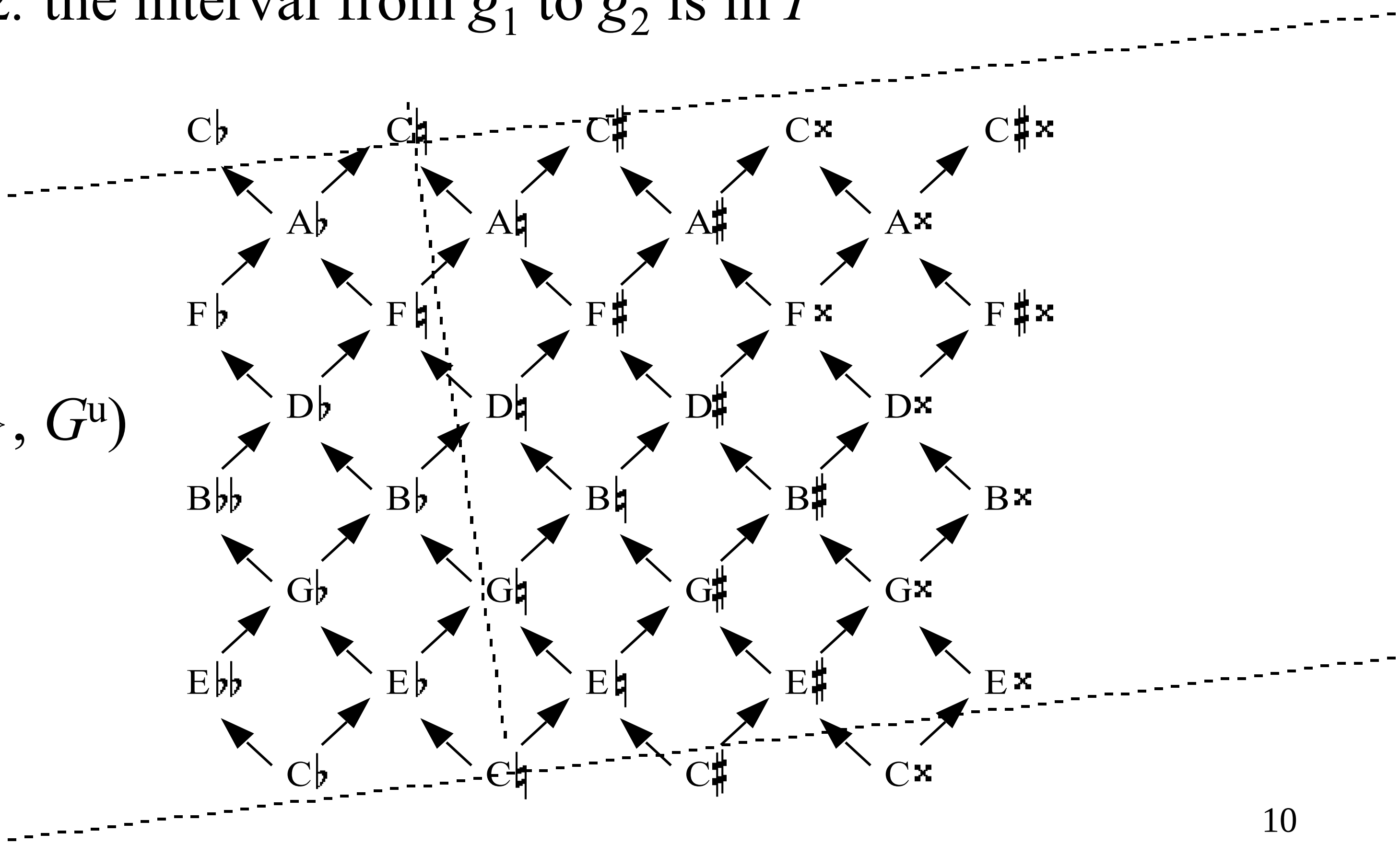
REPRESENTING GENUS RELATIONS USING DIGRAPHS

$$R^{g} = (I\ ,\ G)$$

$g_1\ R^{g}\ g_2$ is true if and only if

1. g_1 and g_2 are both in G
2. the interval from g_1 to g_2 is in I

$R^{g} = (\{(4,2),(3,2)\},\ G^{u})$



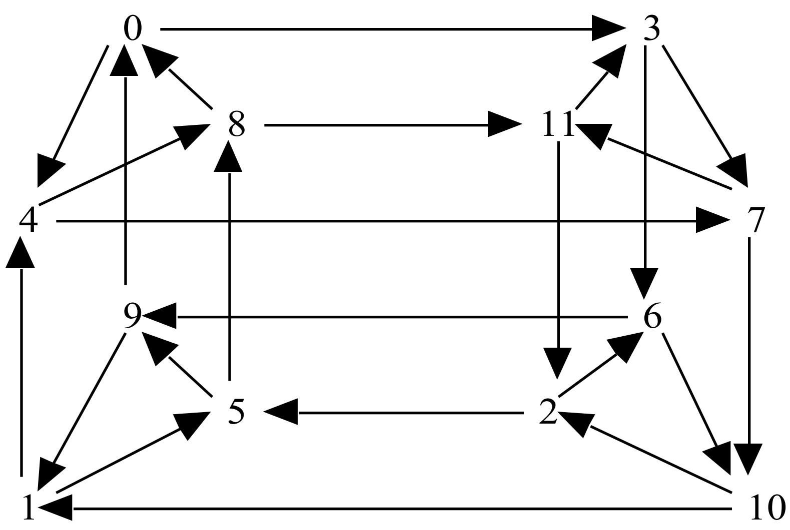
REPRESENTING CHROMA RELATIONS USING DIGRAPHS

$$R^{c} = (I\ ,\ C)$$

$c_1\ R^{c}\ c_2$ is true if and only if

1. c_1 and c_2 are both in C
2. the interval from c_1 to c_2 is in I

$R^{c} = (\{(4,2),(3,2)\},\ C^{u})$



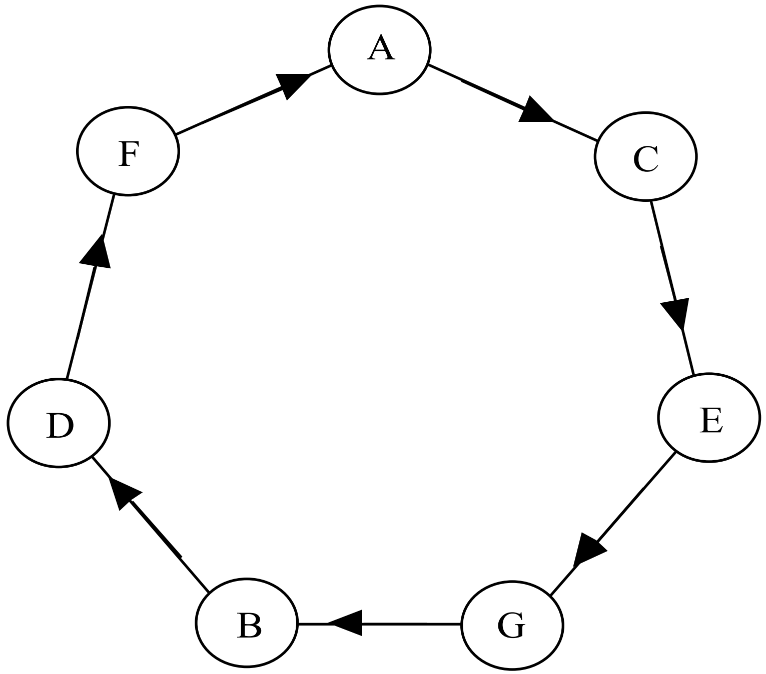
REPRESENTING MORPH RELATIONS USING DIGRAPHS

$$R^{\mathrm{m}} = (I\ ,\ M)$$

$m_1\ R^{\mathrm{m}}\ m_2$ is true if and only if

1. m_1 and m_2 are both in M
2. the interval from m_1 to m_2 is in I

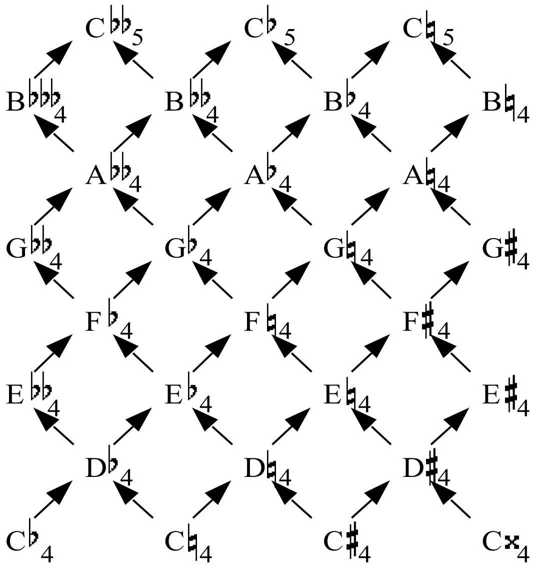
$$R^{\mathrm{m}} = (\{(4,2),(3,2)\},\ M^{\mathrm{u}})$$

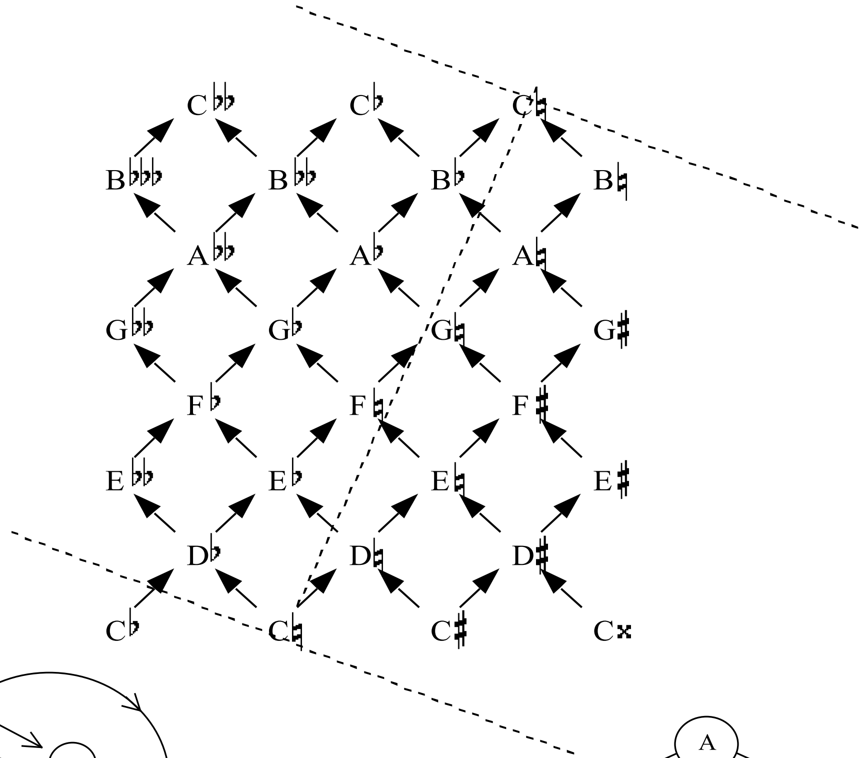


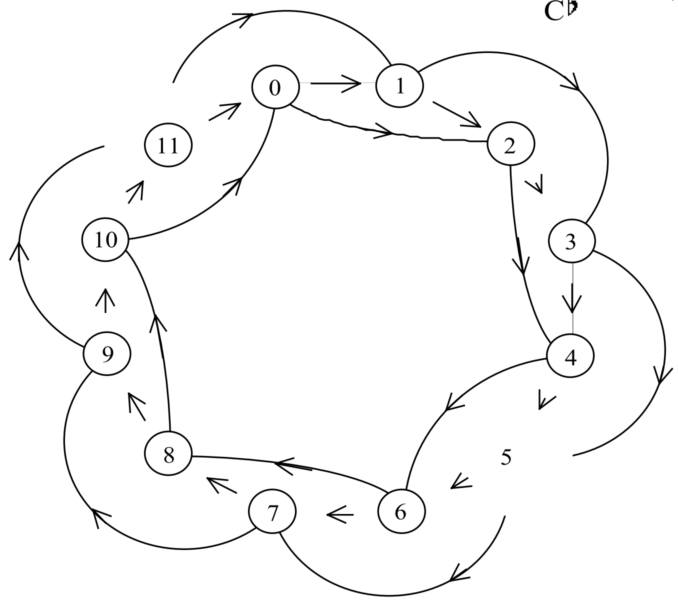
INTERVALS IN THE DIATONIC SET

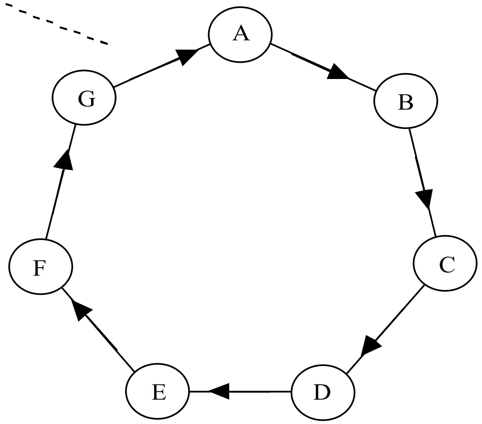
MORPHETIC INTERVAL TRAN EQ CLASS	MORPHETIC INTERVAL TRAN EQ CLASS NAME	CHROMATIC INTERVAL TRAN EQ CLASS	GENERIC INTERVAL TRAN EQ CLASS NAME	GENERIC INTERVAL TRAN EQ CLASS	FREQUENCY OF OCCURRENCE OF GENERIC INTERVAL CLASS IN DIATONIC SET	OCTAVES SPANNED BY CIRCUIT	RELATION PITCH INTERVAL SET
1	SECOND	1	rmi2	(1,1)	2	1	{(1,1),(2,1)}
		2	rma2	(2,1)	5		
2	THIRD	3	rmi3	(3,2)	4	2	{(3,2),(4,2)}
		4	rma3	(4,2)	3		
3	FOURTH	5	rp4	(5,3)	6	3	{(5,3),(6,3)}
		6	ra4	(6,3)	1		
4	FIFTH	6	rd5	(6,4)	1	4	{(6,4),(7,4)}
		7	rp5	(7,4)	6		
5	SIXTH	8	rmi6	(8,5)	3	5	{(8,5),(9,5)}
		9	rma6	(9,5)	4		
6	SEVENTH	10	rmi7	(10,6)	5	6	{(10,6),(11,6)}
		11	rma7	(11,6)	2		

THE "SECONDS" RELATIONS

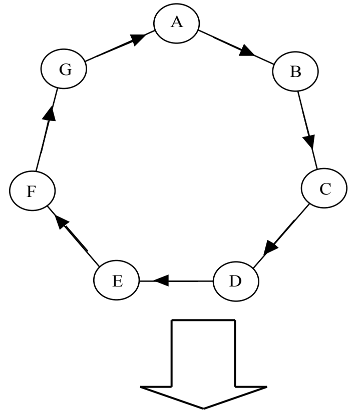








CIRCUITS IN THE “SECONDS” RELATIONS DIGRAPHS



All intervals i*k* must be either rmi2 or rma2
Let say there are n1 rmi2 intervals (Def.1)
and *n*2 rma2 intervals (Def.2)
This implies that *n*1 + *n*2 = 7 (Eq.1)

Must start and end on the same genus therefore must traverse an integer number of octaves. Let’s call this number of octaves n3. (Def.3)

CIRCUITS OF LENGTH 7 IN THIS DIGRAPH

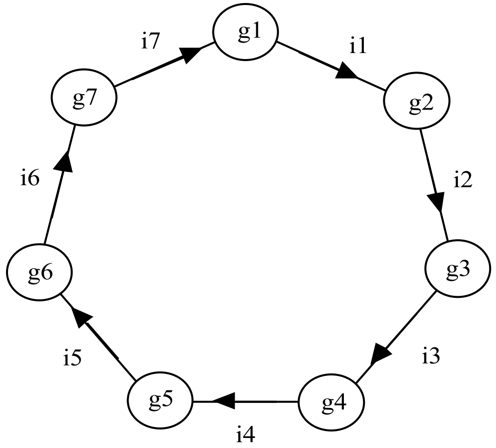
Defs. 1,2 and 3 =>

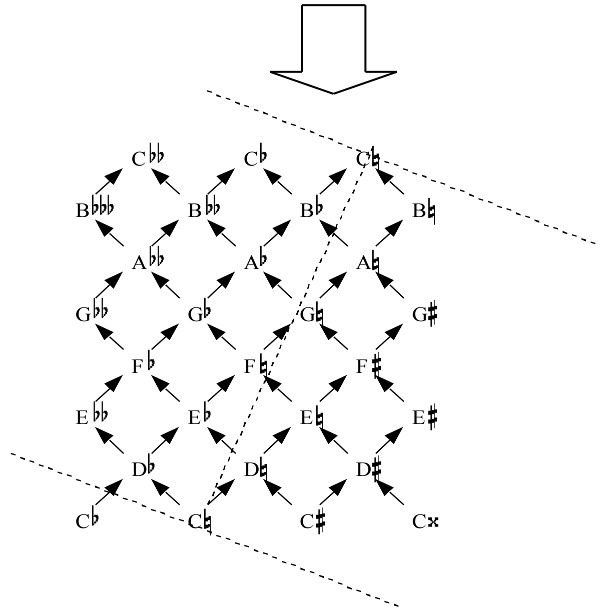
$$(n1 \times 1) + (n2 \times 2) = n3 \times 12 \qquad \text{(Eq.2)}$$
$$(n1 \times 1) + (n2 \times 1) = n3 \times 7 \qquad \text{(Eq.3)}$$

Eq.1 & Eq.3 => 7 = n3 x 7 => <u>n3 = 1</u> (Eq.4)
Eq.4 & Eq.2 => n1 + 2 x n2 = 12 (Eq.5)

Eq.1 => n2 = 7 - n1

Sub in Eq.5:
n1 + 2 x (7 - n1) = 12
=> n1 + 14 - 2n1 = 12
=> <u>2 = n1</u> (Eq.6)
Eq.1 & Eq.6 => <u>n2 = 5</u> (Eq.7)



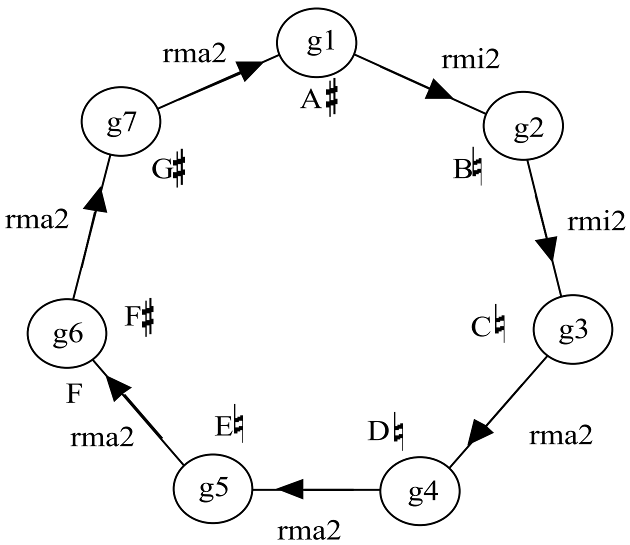


So we’re looking for circuits that contain <u>5 major seconds and 2 minor seconds</u>
(the same as the frequency with which these intervals occur within the diatonic set...)

CIRCUITS IN THE “SECONDS” RELATIONS DIGRAPHS 2

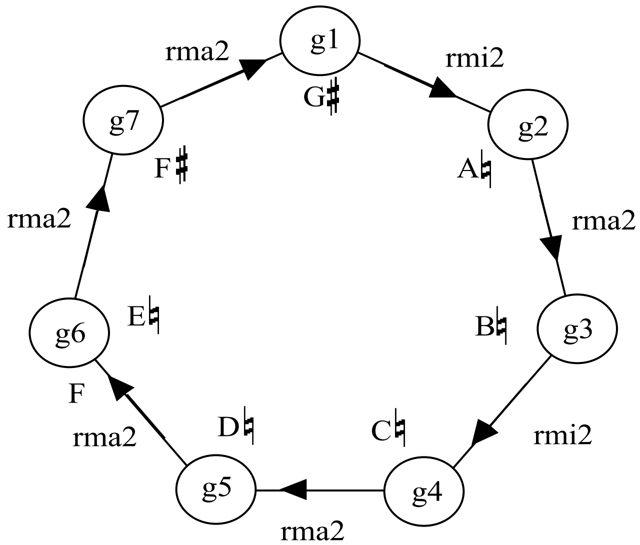
What different orders can these 5 major seconds and 2 minor seconds be placed in?

TWO MINOR SECONDS
CONSECUTIVE



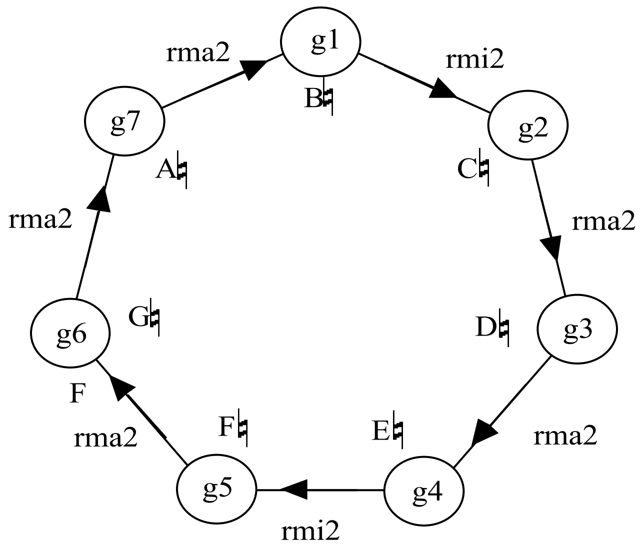
WHOLE-TONE SCALE WITH
MAJOR LEADING NOTE

TWO MINOR SECONDS
SEPARATED BY 1 (AND 4)
MAJOR SECONDS



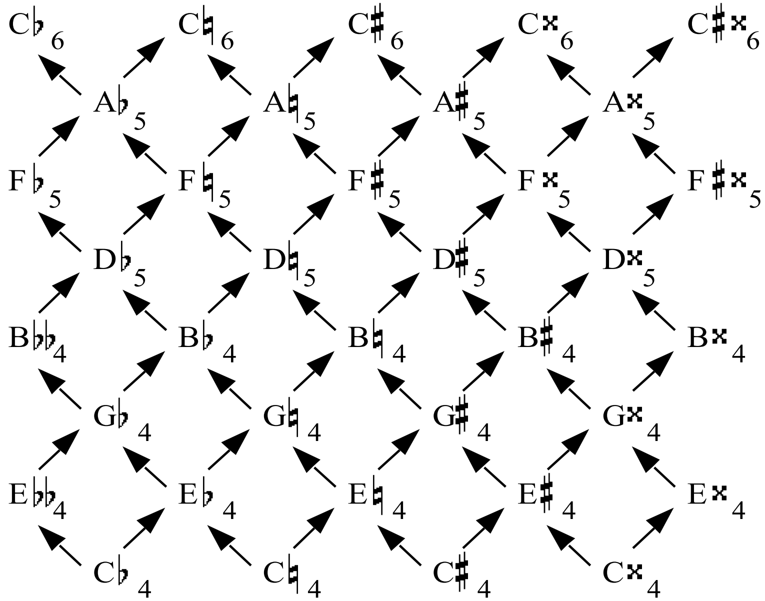
ASCENDING
MINOR MELODIC
SCALE

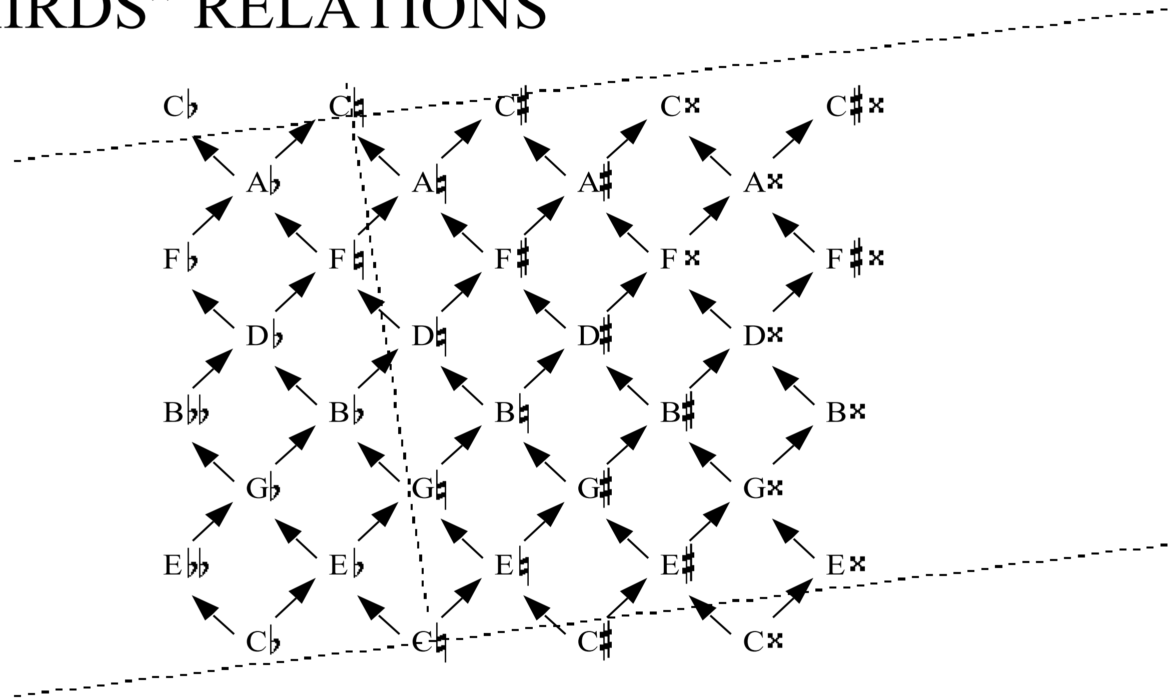
TWO MINOR SECONDS
SEPARATED BY 2 (AND 3)
MAJOR SECONDS

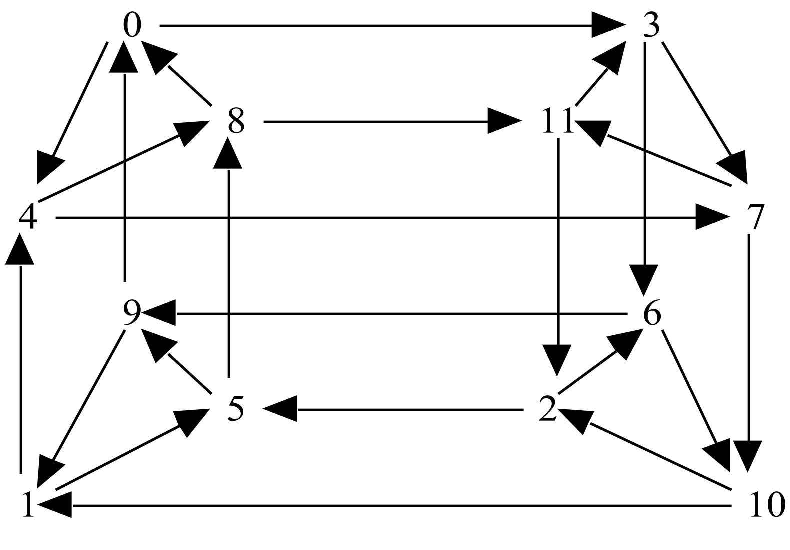


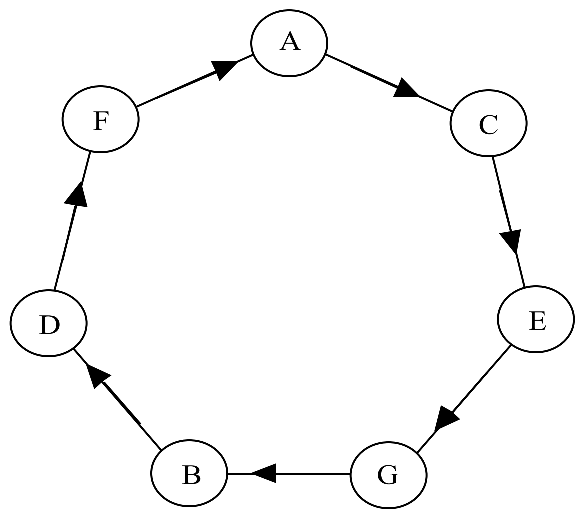
DIATONIC SET

THE “THIRDS” RELATIONS









CIRCUITS IN THE "THIRDS" RELATIONS DIGRAPHS

All intervals *ik* must be either rmi3 or rma3

Let say there are	n1 rmi3 intervals	(Def.1)
and	*n*2 rma3 intervals	(Def.2)
This implies that	$n1 + n2 = 7$	(Eq.1)

Must start and end on the same genus therefore must traverse an integer number of octaves. Let's call this number of octaves n3. (Def.3)

Defs. 1,2 and 3 =>

$(n1 \times 3) + (n2 \times 4) = n3 \times 12$ (Eq.2)

$(n1 \times 2) + (n2 \times 2) = n3 \times 7$

$\Rightarrow (n1 + n2) \times 2 = n3 \times 7$ (Eq.3)

Eq.1 & Eq.3 => $14 = n3 \times 7 \Rightarrow \underline{n3 = 2}$ (Eq.4)

Eq.4 & Eq.2 =>

$(n1 \times 3) + (n2 \times 4) = 24$ (Eq.5)

Eq.1 => $n2 = 7 - n1$

Sub Eq.1 in Eq.5:

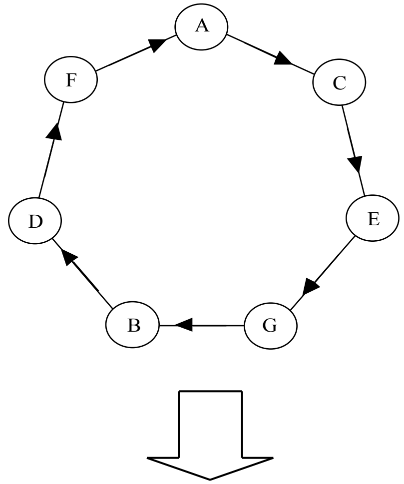
$(n1 \times 3) + 4(7 - n1) = 24$

$\Rightarrow (3 - 4) \times n1 + 28 = 24$

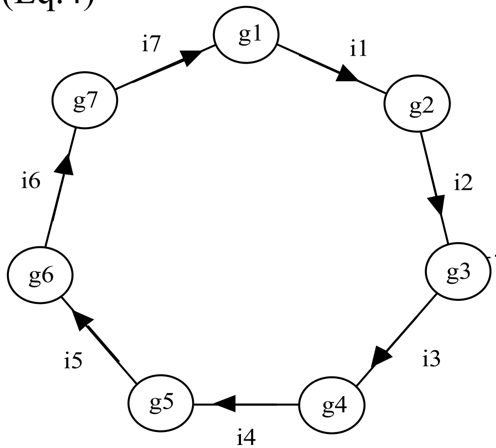
$\Rightarrow -n1 + 28 = 24 \Rightarrow \underline{n1 = 4}$ (Eq.6)

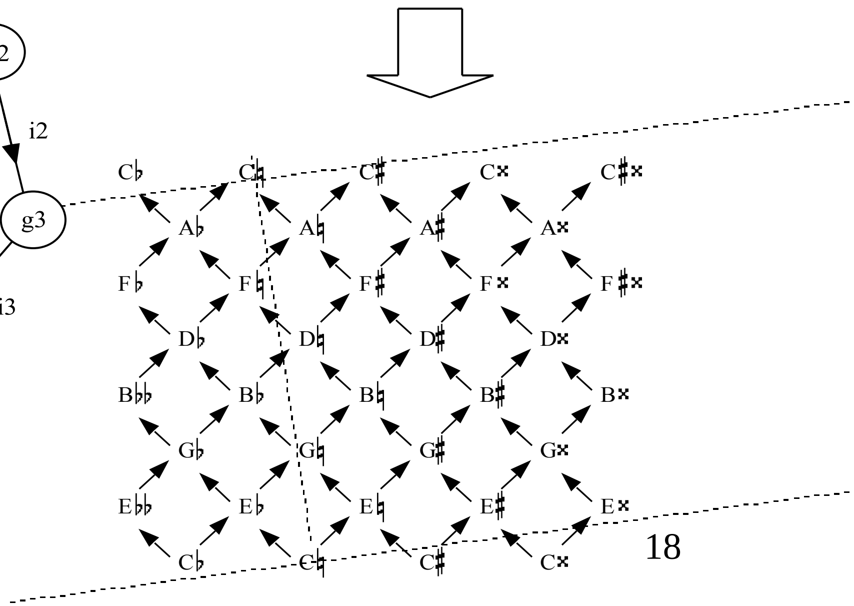
Eq.1 & Eq.6 => $\underline{n2 = 3}$ (Eq.7)

So we're looking for circuits that contain <u>3 major thirds and 4 minor thrids</u>
(again, the same as the frequency with which these
intervals occur within the diatonic set...)



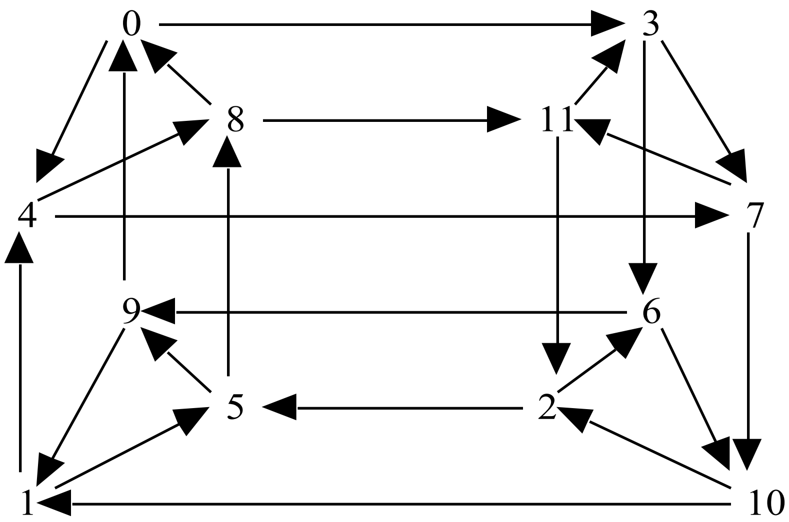
CIRCUITS OF LENGTH
7 IN THIS DIGRAPH





CIRCUITS IN THE “THIRDS” RELATIONS DIGRAPHS 2

WHAT ARE THE DIFFERENT WAYS THAT WE CAN ARRANGE
THE 3 MAJOR THIRDS AND 4 MINOR THIRDS IN THE CIRCUIT?



MUST BE A CIRCUIT IN THE THIRDS CHROMA RELATION DIGRAPH!!!

IF WE PUT <u>3 MAJOR THIRDS</u> CONSECUTIVELY WE COME BACK TO THE INITIAL CHROMA PREMATURELY.

THE SAME HAPPENS IF WE PUT <u>4 MINOR THIRDS</u> IN A ROW

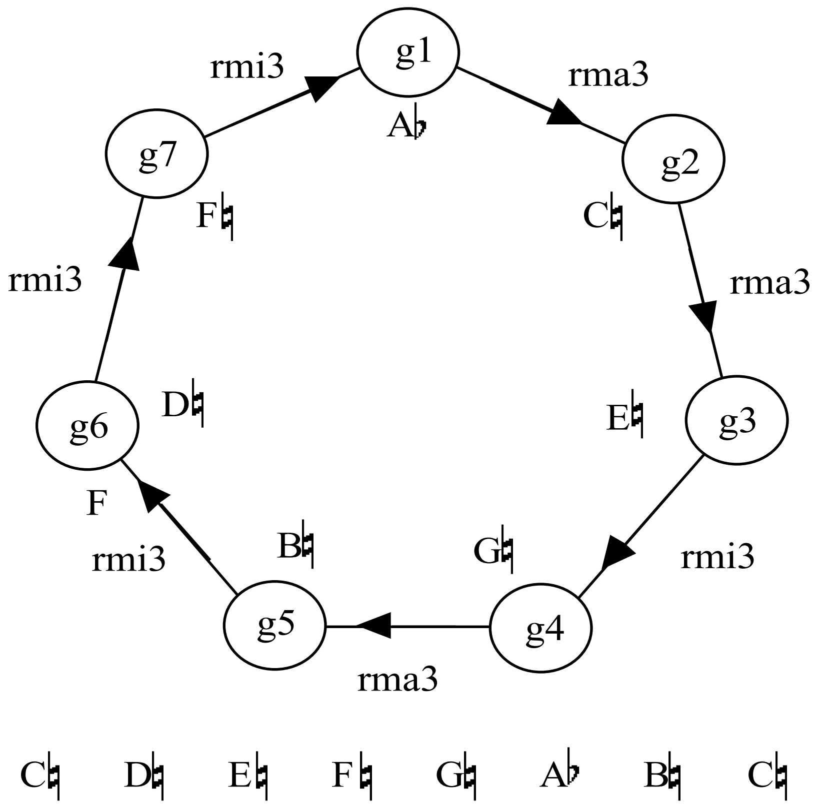
THEREFORE:

1. CANNOT USE MORE THAN <u>2 MAJOR THIRDS</u> CONSECUTIVELY IN THE CIRCUIT
2. CANNOT USE MORE THAN <u>3 MINOR THIRDS</u> CONSECUTIVELY IN THE CIRCUIT.

CIRCUITS IN THE "THIRDS" RELATIONS DIGRAPHS 3

WHAT ARE THE DIFFERENT WAYS THAT WE CAN ARRANGE
THE 3 MAJOR THIRDS AND 4 MINOR THIRDS IN THE CIRCUIT?

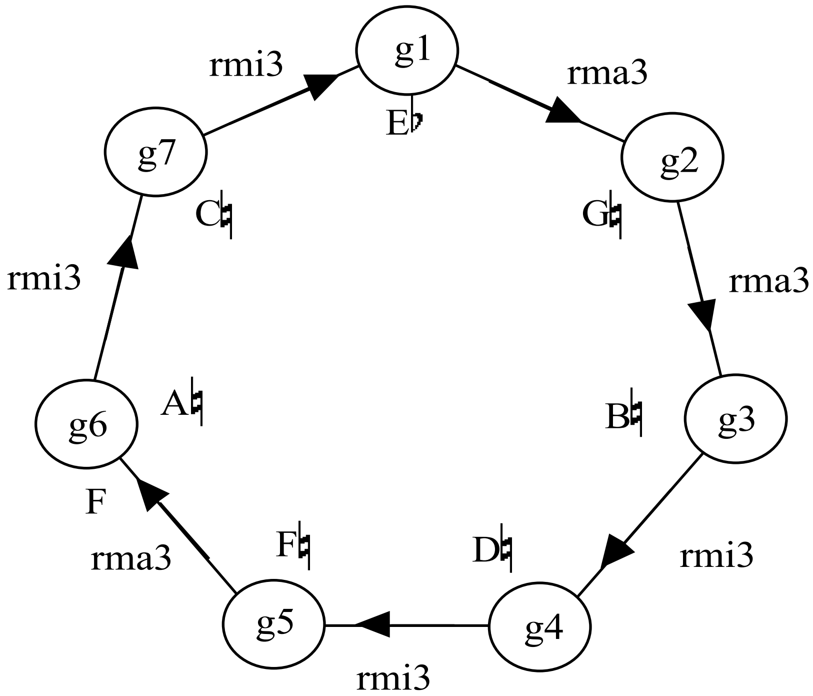
2 MAJOR THIRDS IN A ROW SEPARATED FIRST BY 1 MINOR THIRD THEN BY 3



"MAJOR HARMONIC" SCALE

CIRCUITS IN THE “THIRDS” RELATIONS DIGRAPHS 4

2 MAJOR THIRDS IN A ROW SEPARATED FIRST BY 2 MINOR THIRD THEN BY 2

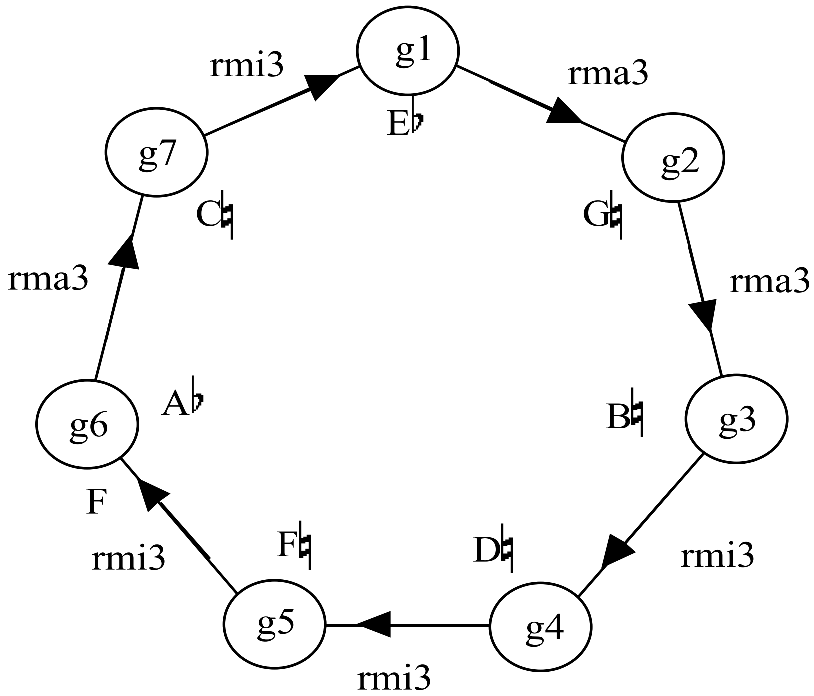


C♮ D♮ E♭ F♮ G♮ A♮ B♮ C♮

ASCENDING MINOR MELODIC SCALE

CIRCUITS IN THE “THIRDS” RELATIONS DIGRAPHS 5

2 MAJOR THIRDS IN A ROW SEPARATED FIRST BY 3 MINOR THIRD THEN BY 1

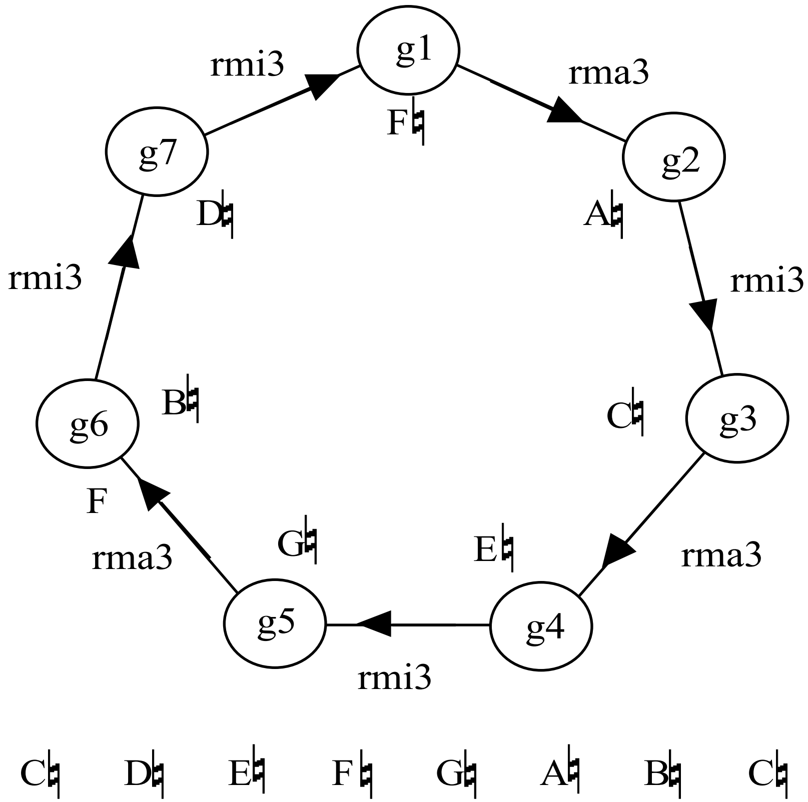


C♮ D♮ E♭ F♮ G♮ A♭ B♮ C♮

MINOR HARMONIC SCALE

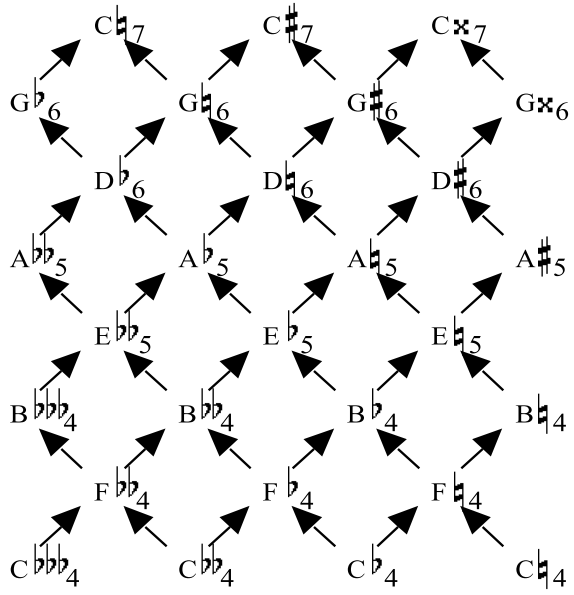
CIRCUITS IN THE “THIRDS” RELATIONS DIGRAPHS 6

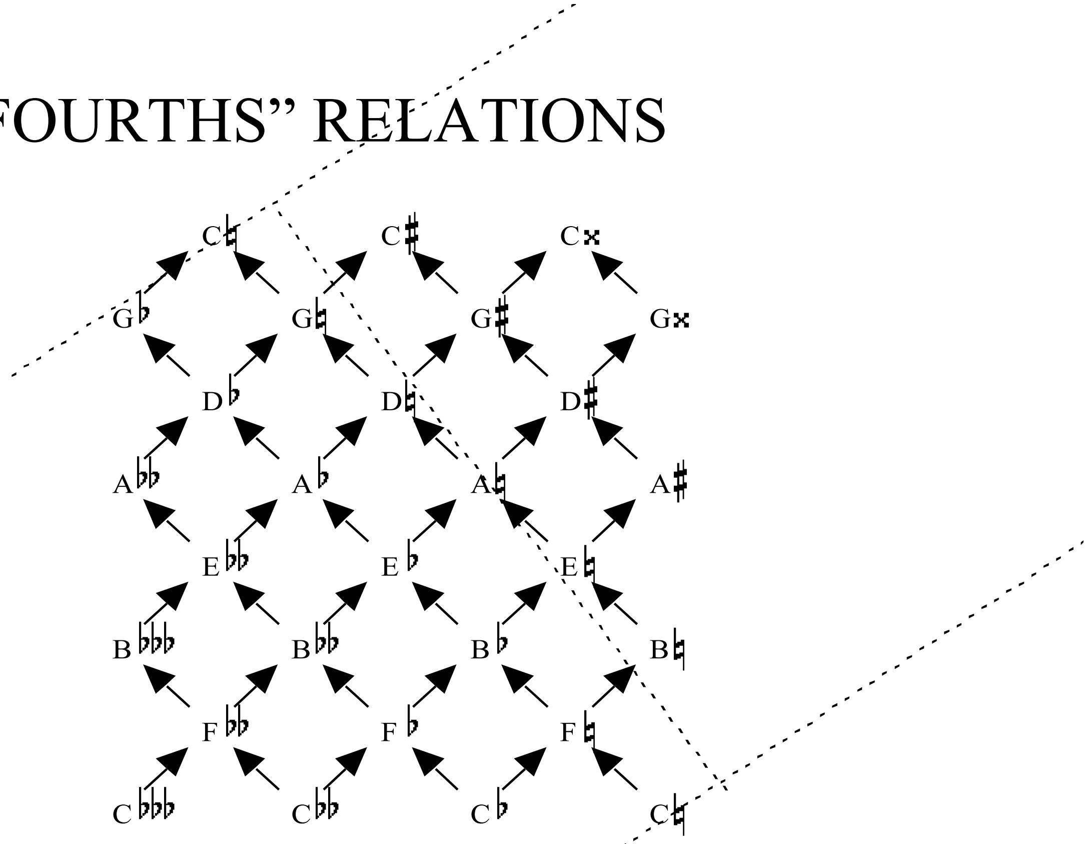
ONE MAJOR THIRD CONSECUTIVELY

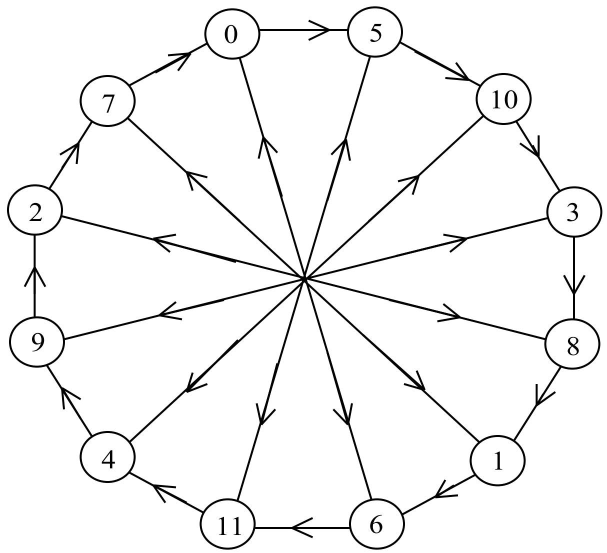


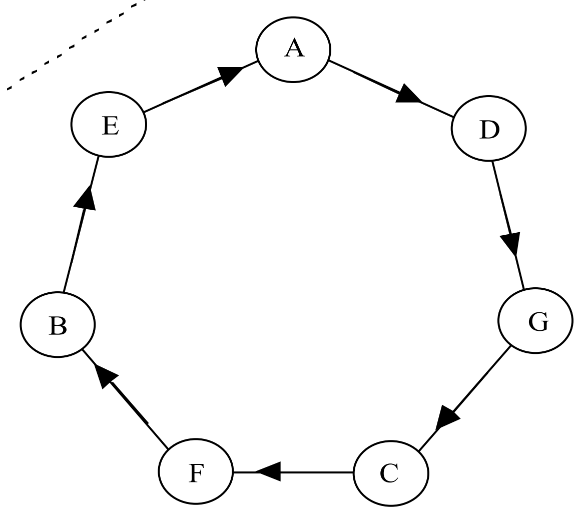
DIATONIC SET

THE “FOURTHS” RELATIONS

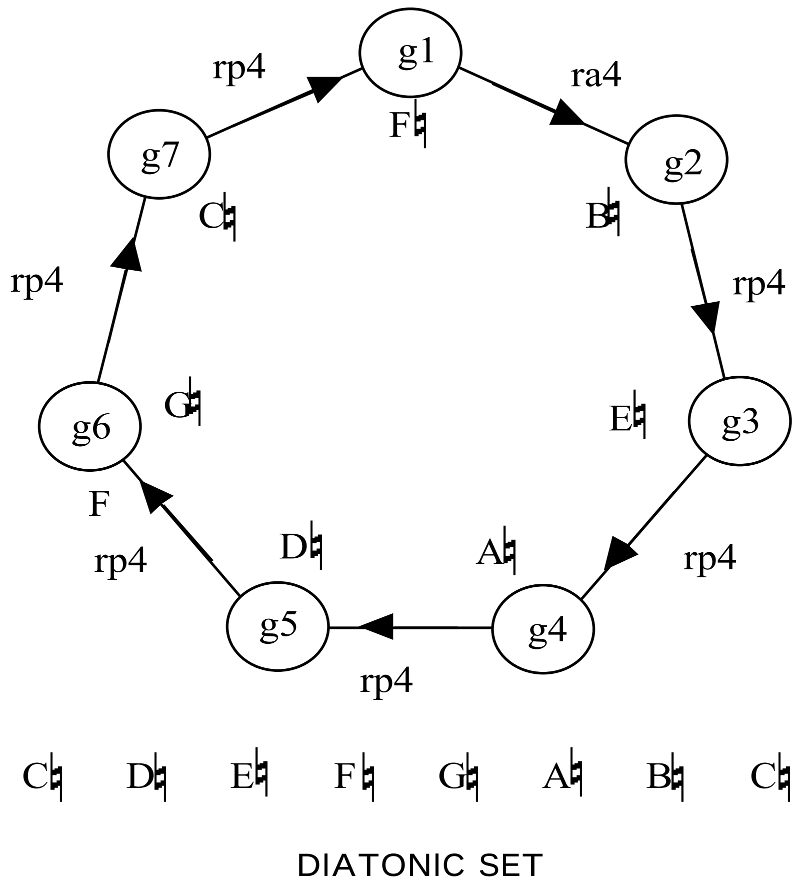








CIRCUITS IN THE "FOURTHS" RELATIONS DIGRAPHS



CONCLUSIONS

CIRCUITS IN THE SECONDS(AND SEVENTHS) GRAPHS

WHOLE-TONE WITH LEADING-NOTE
ASCENDING MINOR MELODIC SCALE
DIATONIC SET

CIRCUITS IN THE THIRDS (AND SIXTHS) GRAPHS

MAJOR HARMONIC SCALE
DIATONIC SET
MINOR MELODIC SCALE
MINOR HARMONIC SCALE

CIRCUITS IN THE FOURTHS (AND FIFTHS) GRAPHS

DIATONIC SET

Using digraphs to represent pitch relations

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by

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Slide 1

- I'm going to talk to you for about half an hour about using mathematical objects called digraphs to represent pitch relations.
- When use digraphs to represent concepts in tonal theory like scales, chords and keys you see that these concepts have some very special properties that you don't see when you look at them from any other representational perspective.
- For example, the tonal scales – the diatonic major, the harmonic minor and the melodic minor – are revealed to possess some special properties that no other pitch collections possess and the diatonic pitch collection turns out to be very special indeed.

Slide 2

- What is a digraph anyway?
- One of these things on the left here. It's basically collection of points called *vertices* joined together in some way with directed lines called *arcs*.
- I've labelled the vertices with the numbers 1 to 6 and the arcs with the letters a to i.
- Now the whole point of digraphs is that you can go for little trips round them.
- For example, we could start at 1, go along *a* to 2 then go around *b* and end up back at 2 again and then go down to 5 via *f*.
- Second walk.
- Any trip like this around the digraph is called a *walk*.
- Every walk has an initial vertex, a terminal vertex and a length. Examples.
- Every walk also has a walk set. Examples.
- Digraph theory recognizes some special kinds of walk:
    - Trails
        - Define and give example.
        - Initial vertex, terminal vertex, length, walk set.
        - Second walk example not a trail because it passes along arc *c* twice.
    - Closed trail
        - Define and give example.
    - Path
        - Define and give example.
    - Circuit
        - Most special type of walk.
        - Define and give example.
        - Difference between circuits and closed trail example.

Slide 3

- Talk now about various different ways of representing pitch.
- We've got a staff here with a number of notes on it.
- Each note has a pitch associated with it.
- We can represent that pitch in a number of different ways.
    - Standard A.S.A. pitch name
        - Each pitch name composed of a letter-name, an accidental and an octave number. Example.
        - Letter-name defined by position on the staff. Example.
        - Accidental determined by explicit symbols or the key-signature (except that assumed to be natural if left out). Example.
        - Octave number is the same as the next C below it on the staff.
            - Means that the fact that a note's octave number is higher than that of another does not mean that its sounding pitch is higher. Example.
        - The accidental can be any number of sharps or flats.
            - Ridiculous C here with 8 sharps.
            - Logic of pitch-naming system allows this.
            - Quite feasible (if unlikely) that in order to correctly notate some passage of tonal music, one would have to use more than the usual maximum of 2 sharps or flats.

Slide 4

- I can give you an example here.
- At the top we have a chord sequence that modulates from C major to F sharp major.
- [Play it on piano.]
- At the bottom we have the same chord sequence transposed up a minor second so that it starts in C sharp major and ends in F double-sharp major.
- As you can see, in order to notate this second passage correctly we have to use triple sharps.
- So it is feasible that that one might have to use any number of sharps or flats to correctly notate a passage of tonal music.

Slide 3

- On the second line here I've put the letter-name representation of the pitch of each note.
- Letter-names are not to be despised for their simplicity because they tell us something quite important about the function of a note within its tonal context.

Slide 5

- In fact, in a correctly notated score of a piece of tonal music, if two notes have the same letter-name in the same tonal region in the music, this means that both are functioning as the same scale-degree in the key that operates at that point in the music.
- Here, for example, in the upper score both of these notes are functioning as leading-notes in G major so they both have to be spelt as Fs.

- If we spelt the second one as something else – as a G flat, for example – that would be incorrect because it implies that this note has a different scale degree from this note in the key that operates at this point in the music.
- But this note is functioning as a leading-note in G major therefore we have to spell it as an F sharp and nothing else.
- So don't despise letter-names.

Slide 3

- The third row shows a numeric representation that I call *morph* which is simply a mapping of the letter-names onto the numbers 0 to 6 so that A maps onto 0, B maps onto 1, C maps onto 2 and so on.
  - This lets us do clock arithmetic modulo 7 on the letter-names.
- On the fourth row I've put what I call *genus-name*.
  - This is simply the pitch-name without the octave indication so that it represents octave-equivalence.
- The fifth row shows the *genus* of each note.
  - This is a numeric representation of the genus-name so that we can do computations on genus-names.
  - A *genus* is an ordered pair in which the first element is the morph (which as we've already discussed is a numeric representation of the letter-name) and the second element is the *displacement* which is a numeric representation of the accidental.
    - So a flat is represented by –1, a natural by 0, a sharp by 1 and so on.
- The next line shows the *chroma* of each note.
  - This is the same as the normal idea of pitch class where each sounding pitch is represented by the same number as every other sounding pitch that is an integer number of octaves away from it. Example – Cn4, Cn5, Bs3 all represented by 3.
  - Notice though that I use the chroma 0 to represent A natural rather than C natural.
  - Because in the equal-tempered system the octave is normally divided up into 12 different sounding pitches each a semitone apart the value of chroma ranges from 0 to 11.
- The next line shows what I call *chromatic pitch*.
  - This is a numeric representation of sounding pitch that does not represent octave equivalence.
  - So the bottom A natural on the piano keyboard is assigned a chromatic pitch of 0, the B flat or A sharp just above that bottom A is represented by 1, and the G sharp just below the bottom note on the piano would be represented by –1.
  - So in general if you rise by a semitone, the chromatic pitch increases by one and if you fall by a semitone the chromatic pitch falls by one.
- The morphetic pitch of a note which I've shown on this line here, represents the vertical position of the note on the staff.
  - So again, a note representing the bottom A natural on the piano keyboard is represented by a zero. This means that middle C has a morphetic pitch of 23, which means that any D in the space just above will have a morphetic pitch of 24 and so on.

- Finally on the bottom line I've put a numeric representation that I call simply *pitch* from which all these other representations – chroma, morph, genus and so on - can be calculated algorithmically.
  - A pitch is simply an ordered pair in which the first element is the chromatic pitch and the second element is the morphetic pitch.

Slide 6

- I'm now going to talk for a couple of minutes about the idea of a *pitch interval* and the idea of *generically equivalent pitch intervals*.
- For the moment just look at the bottom half of this slide.
- As you can see, we have a number of notes and we can represent the intervals between these notes in essentially two ways:
  - We can use pitch interval names like “Rising major third”, “Falling diminished diminished fourth”
  - And we can use a numeric representation that consists of an ordered pair of integers in which the first element of the pair (which we call the *chromatic interval*) represents the change in chromatic pitch and the second element of the pair (which we call the *morphetic interval*) represents the change in morphetic pitch.
  - Work through examples.

Slide 7

- Now explain idea of a generic equivalence class of pitch intervals.
- Read definition.
- Give example using pitches and intervals on slide.
- Represent generic equivalence class using the interval whose morphetic interval is between 0 and 6.

Slide 6

- Show more examples of generic equivalence classes using intervals on slide.

Slide 8

- Now introduce you to the concept of a *pitch relation*.
- You specify a pitch relation by specifying a set of intervals called the pitch relation interval set and set of pitches called the pitch relation pitch set.
- You use pitch relations to express logical propositions of the form shown.
- Talk about the examples.

Slide 9

- Now show you how we can represent pitch relations using digraphs.
- Represent each member of the pitch set by its own vertex. See example.
- This is just a small portion of the complete graph – the universal pitch set is infinite so this graph is actually an infinite graph.

- Draw an arc between any two pitches if and only if the interval from the first to the second is in the pitch interval set. See example.
- Gives us a regular infinite planar graph in which all the walks are paths.

Slide 10

- On an analogy with pitch relations, we can define the idea of a genus relation and we can represent these genus relations using digraphs.
- A genus relation is an ordered pair in which the first element is a pitch interval set again and the second element is a genus set.
- See definition slide.
- See slide for how to represent genus relations using digraphs.
- Gives infinite cylindrical graph for rising major and minor third relation.

Slide 11

- Just as for pitch and genus relations, so we can have chroma relations and we can represent them using digraphs too.
- Chroma relation is an ordered pair in which the first element is a pitch interval set and the second element is a chroma set.
- See definition on slide.
- See example of representing this relation on slide.
    - For thirds graph, gives structure that can be embedded in the surface of a torus.
    - Closely related to Balzano's C3 x C4 direct product lattice which was in fact the thing that gave me the idea for all of this.

Slide 12

- Finally can define the concept of a morph relation and represent them using digraphs too.
- Define concept and give example using slide.
- For a given pitch interval set, we can define a pitch relation, a genus relation, a chroma relation and a morph relation and we can represent these relations using digraphs.

Slide 13

- Table shows the generic equivalence classes of intervals that occur within the diatonic set.
- We're going to ignore unisons.
- All 6 of the possible morphetic interval classes that can occur – seconds, thirds, fourths etc.
    - Each of these morphetic interval classes occurs 7 times.
- Every chromatic transpositional equivalence class occurs within the diatonic set.
- For each morphetic interval class there are two chromatic interval classes that differ in size by just one semitone.
- Each inversional equivalence class of chromatic interval occurs with a different frequency within the diatonic set and all the frequencies between 1 and 6 are represented.
- We'll look at the pitch, genus, chroma and morph relation digraphs for these interval sets here.

- We only need to look at the graphs for the seconds, thirds and fourths because the graphs for the fifth relations are just the same as for the fourths but the arrows are pointing in the opposite direction. Similarly for the sixths and sevenths.

Slide 14

- Start by looking at the graphs that represent the pitch, genus, chroma and morph relations whose pitch interval set contains the rising major and minor seconds.
- Look at each graph
    - Topology
    - Circuits etc.
- Corresponding paths within the four graphs.
- Going to talk about those paths in the pitch relation graph such that the corresponding walks in the genus, chroma and morph relation graphs are all circuits.

Slide 15

- What kind of circuits do we end up with?
- Start with morph graph limits length to 7.
- Represent circuit generally.
- Go through maths.

Slide 16

- Just use the slide.

Slide 17

- Digraphs that represent the pitch, genus, chroma and morph relations whose pitch interval set contains just the rising major and minor thirds.
- Topology.
- Will look at paths whose corresponding paths are circuits in the genus, chroma and morph relation digraphs.

Slide 18

- Start with morph digraph restricting length to 7.
- Then just say that by the same argument as before have to have 3 major thirds and 4 minor thirds

Slide 19-23

- Just use slides.

Slide 17

- So if we want walks in the pitch relation digraph whose corresponding walks in the genus chrom and morph relation digraphs are circuits, we will always end up with either a diatonic set, a minor harmonic scale, a minor melodic ascending scale or a major harmonic scale.

Slide 24

- Finally, we can do exactly the same kind of thing for the fourths graphs.
- Here are the graphs, we want to find circuits and we find that there is only one type of circuit in this genus relation graph that satisfies that.
- It is a circuit that consists of one augmented fourth and six perfect fourths (again the same frequency as that with which these intervals occur in the diatonic set) and we find that all circuits of length seven in this graph have walk sets that are diatonic sets.

Conclusion

- If we look at the intervals that occur within the diatonic set, and define relations and look at the digraphs that use those intervals as their interval sets, we find (SEE SLIDE)